The Dividend Discount Model in the Long-Run: A Clinical Study

By:
Stephen R. Foerster and Stephen G. Sapp*
Richard Ivey School of Business
The University of Western Ontario
London, Ontario, Canada, N6A 3K7

First Version: February 26, 2004
Current Version: September 18, 2005

Forthcoming *Journal of Applied Finance*

Abstract

Finance professionals frequently value assets using fundamental valuation methods which discount the expected cash flows received by investors. Using information on the share price, dividend payments and earnings for a single firm over a period of more than 120 years, we compare the actual share price to the expected price—calculated using several of the most commonly used fundamental valuation methods. Since these methods depend on the estimation of inputs—such as the discount rate and growth rate—we discuss the sensitivity of the expected prices to different estimation techniques and the relevant assumptions across various economic conditions. Over our entire sample period, we find that dividend-based models perform well at explaining actual prices; they perform better than commonly used earnings-based models (such as the Fed Model).

JEL Codes: G21, G35
Keywords: Dividend Discount Model, Valuation

---

*Paul Desmarais/London Life Faculty Fellowship in Finance (Foerster) and MBA Class of 1989 Faculty Fellowship (Sapp). The research assistance of Jarred Freeman is gratefully acknowledged. We thank an anonymous referee, Jerry Mulcahy, Rick Robertson, Sergei Sarkissian, Ken Snowden and seminar participants at McGill University and the University of Western Ontario for useful comments, and the Social Sciences and Humanities Research Council for financial support. Please address all correspondence to Steve Foerster: Richard Ivey School of Business, The University of Western Ontario, London, Ontario, Canada, N6A 3K7; Phone: 1-519-661-3726; Fax: 1-519-661-3485; E-mail: sfoerster@ivey.uwo.ca.

The Dividend Discount Model in the Long-Run:
A Clinical Study

Abstract

Finance professionals frequently value assets using fundamental valuation methods which discount the expected cash flows received by investors. Using information on the share price, dividend payments and earnings for a single firm over a period of more than 120 years, we compare the actual share price to the expected price calculated—using several of the most commonly used fundamental valuation methods. Since these methods depend on the estimation of inputs—such as the discount rate and growth rate—we discuss the sensitivity of the expected prices to different estimation techniques and the relevant assumptions across various economic conditions. Over our entire sample period, we find that dividend-based models perform well at explaining actual prices; they perform better than commonly used earnings-based models (such as the Fed Model).

JEL Codes: G21, G35
Keywords: Dividend Discount Model, Valuation

Introduction

The fundamental value of an asset can be viewed as a function of three variables: the size, timing and uncertainty of the cash flows the asset will generate for investors over its lifetime. For equities, the cash flows are generally dividends and the uncertainty lies in the timing and growth of the firms' earnings and its subsequent ability to pay dividends. Since dividends historically have depended on the size and sustainability of earnings, both dividends and earnings are key determinants of the value of equity. For example, Lintner (1956) interviewed managers from 28 companies to determine how and why their firms paid dividends. He found that managers target a long-term payout ratio (dividends as a percentage of earnings). Even though recent studies, such as Brav et al. (2005), suggest that managers now focus on maintaining a steady growth rate of dividends rather than a consistent payout ratio, dividends and earnings continue to play a key role in discussions (among both academics and practitioners) about how to value equity. In this paper, we provide a detailed evaluation and comparison of several commonly used methods for the valuation of equity in order to explore their performance over time and various economic conditions.

To most effectively investigate how different factors influence these valuation techniques, we study data on a single firm which has regularly paid dividends over a long period. (The firm studied has, to our knowledge, the longest continuous available dividend stream of any North American firm.) The first set of models we consider are versions of the standard dividend discount model. Though some empirical studies find that investors consider dividends when valuing assets (e.g., Fama and French (1988) or for a survey see Allen and Michaely (2002)), there is little consensus over longer periods of time (e.g., Goyal and Welch (2003)). The next set of models we study are based on earnings, such as the well known "Fed Model."

Focusing on only one firm, we recognize that there is an issue of generalizability; thus, our results are not meant to present a definitive statement about the applicability of these models to the pricing of *all* assets. There is, however, an important corollary: if the dividend discount model and earnings-based models do *not* appear to be appropriate to the one firm to which they should have the greatest chance of applying, then these models may be of questionable value to practicing managers. To investigate the robustness of these valuation techniques for changing economic conditions and different assumptions, we compare their performance and their sensitivity to the assumptions used when implementing them in practice over periods characterized by a wide variety of economic conditions.

We focus on the Bank of Montreal because it is historically one of the most preeminent banks in North America and has consistently paid a regular dividend.[1] This status allows us to study the earnings and dividend payout behavior of a firm in the same industry over a very long period of time. The payment of a regular dividend is important since many of the valuation techniques we consider are based on the fundamental concept that a firm is worth the discounted value of the future cash flows (i.e., dividends) one would expect to receive from holding its equity.[2] The motivation for the use of dividend-based models in valuing equity is well expressed in the following quote from Williams (1938):

> "…[A] stock is worth the present value of all the dividends ever to be paid upon it, no more, no less…Present earnings, outlook, financial condition, and capitalization should bear upon the price of a stock only as they assist buyers and sellers in estimating future dividends."

Formally, the dividend discount model states that the price for an asset is the value of all the future payments it is expected to provide discounted at the appropriate rate. Building on this definition, Gordon and Shapiro (1956) and Gordon (1962) present a special case of the general model—often referred to as the Gordon growth or constant growth model. In this model, the value of the firm's equity can be represented as a growing perpetuity based on next period's expected dividend. Because the estimation of the future growth rate of dividends required by this model is less stringent than the estimation of all future dividend payments, this model is very popular in practice. Though much of our discussion focuses on these two models, we realize that there are many alternative models. Recognizing the important role of

---

[1] Historical information on the Bank of Montreal is provided in the Appendix.

[2] We recognize that dividends are not the only way of returning value to shareholders. Studies such as Bagwell and Shoven (1989), Grullon and Michaely (2002) and Brav et al. (2005) discuss the increasing prevalence of share buybacks and other means to return value to shareholders. This is a further advantage of exemplifying Bank of Montreal: it only used share buybacks in 1995, 1996, 2000 and 2001—and correcting for these does not impact our findings.

earnings in these relationships, we also include earnings-based models (such as the Fed Model). Earnings are related to the future returns investors can expect (i.e., a firm's dividend paying ability).

To evaluate these valuation techniques, we compare the observed price at each point in time to the expected price obtained using the dividend discount model (DDM) and the Gordon growth model (GGM). The DDM uses perfect foresight regarding the future dividend payments and uses the capital asset pricing model (CAPM) to estimate the discount rate. Since perfect foresight of dividends is unlikely, we investigate the performance of the GGM, which only requires one to forecast the future growth rate of dividends. We find that both models perform well at explaining observed prices under some of the most commonly used assumptions. Relaxing our assumption that it is only dividends that matter, we consider the Fed Model, which compares the firm's earnings yield (the level of earnings compared to the price) to the yield on a long-term government bond. Although we find a strong correlation between the earnings yield and the yield on the government bond, the model has a weak level of predictability for the actual stock price. Overall, we find that dividend-based models perform best.

Although our models appear to provide reasonable estimates for the value of the asset under the most common assumptions, we investigate the limitations of the different techniques and their sensitivity to the necessary assumptions when applying them in practice. Building on previous studies that discuss issues related to estimating the expected equity premium or discount rate (e.g., Jagannathan, McGratten and Scherbina (2000), Welch (2000), Claus and Thomas (2001), Pastor and Stambaugh (2001), Arnott and Bernstein (2002) and Fama and French (2002)), we find that the equity premium and investors' expectations regarding the equity premium have changed significantly over time. Overall, the implied costs of equity appear to be lower than the estimated or expected costs of equity. The apparent exceptions are the several years between World Wars I and II (mainly for the implied cost of equity using the GGM) and from 1975 to the end of the sample. We also consider the suggestion that investors undervalue equities in the presence of inflation by using an incorrect discount rate (e.g., Modigliani and Cohn (1979)). Although recent studies, such as Ritter and Warr (2002), have found evidence supporting this hypothesis in the 1980s, we do not find strong evidence of this throughout our sample. The other important input is the dividend growth rate. Interestingly, the growth rate in the economy as measured by the change in the Gross National Product (GNP) is one of the best performing estimates over our entire sample—even though it ignores many of the other macroeconomic-based factors which can influence firms' abilities to pay dividends (e.g., Booth (1998)).

The paper is organized as follows. Section 2 presents the different valuation models; our data are described in Section 3. The results are presented in Section 4 and Section 5 concludes.

2. Empirical Models and Hypotheses

To determine the expected price of an asset, we assume that investors use fundamental valuation techniques. The first model we consider is the dividend discount model (DDM). The DDM is the most commonly used fundamental valuation model in practice. It estimates the value of a common share at time t using the relationship:

$$E_t[\mathrm{P}_t] = \sum_{i=t+1}^{\infty} E_t\left[\mathrm{D}_i\right]/(1+r_t)^{i-t} \ , \qquad (1)$$

where $E_t[P_t]$ is the expected intrinsic value or price that we would expect to pay for the share in year t based on the information we have at time t; D_i is the nominal annual dividends we expect to be paid on the share at time i; and r_t is the discount rate investors demand at time t.[3] Because we do not have the required infinite amount of information on the dividends, we operationalize this model by valuing the firm in two stages. The first stage determines the value based on the period for which we have the actual dividend information and the second is a terminal value—the last price from our sample:

$$E_t[\mathrm{P}_t] = \sum_{i=t+1}^{T} E_t\left[\mathrm{D}_i\right]/(1+r_t)^{i-t} + E_t\left[\mathrm{P}_T\right]/(1+r_t)^{T-t} \ . \qquad (2)$$

[3] Throughout the paper we interchangeably refer to this as: the required return; cost of equity; or simply the discount rate.

Assuming investors use the expected dividends and expected terminal value at time T to estimate the intrinsic value at each point in time, this representation captures the effect of both expected changes in dividends and capital gains on current prices. In an efficient market, this should equal the current price.

It is important to recognize that this model requires as inputs: estimates of the future dividends; the terminal stock price; and the appropriate discount rate—all of which should be determined using only information available at each time t. For our analysis, we start with the assumption that investors' best guesses of the anticipated dividends and the terminal value are the actual dividends paid over our sample period and the actual "terminal" price (in our case as of 2003); therefore, we need only to estimate the discount rate or cost of equity, r_t. We estimate this using one of the most commonly used techniques in practice: the Capital Asset Pricing Model (CAPM):

$$r_t = r_{f,t} + \beta_{BMO}(MRP_t), \qquad (3)$$

where $r_{f,t}$ is the yield on a short-term government bond;[4] β_{BMO} is the beta of Bank of Montreal (ticker symbol BMO) assumed to be 1.0;[5] and MRP_t is the expected market risk premium. The expected MRP at time t is calculated using two standard techniques. First, we calculate the MRP on a rolling 30-year basis, from the return on the U.S. market less the return on a short-term government bond.[6] Second, we consider an expanding window starting from our first observation up to time t. Both methods provide us with market premiums that could have been used by market participants at time t and that allow us to determine the sensitivity of our results to two commonly used means for estimating the discount rate.

---

[4] We use the short-term yield to be consistent with our revaluation of the asset every year. However, we also consider the use of the yield on a long-term government bond, and the results are similar. They are therefore not presented.

[5] This appears to be a reasonable assumption given the size and importance of banks over this period as well as the size and importance of the Bank of Montreal. The large role played by banks in the Canadian market is evidenced by their approximate 50% share of the total market capitalization around 1900 and about 30% as of 2003. Nevertheless, we do consider the sensitivity of our estimates to changes in the assumptions on the value of β_{BMO}.

[6] Although we would ideally use Canadian data, Booth (2001) among others suggest that U.S. market data may be more reliable, especially historically, due to the increased depth and liquidity of the U.S. markets. Examining correlations across the U.S. and Canadian data we find that the average correlations for equity markets and fixed income markets are frequently above 95% suggesting that both markets react similarly to changing economic conditions and the use of U.S. values should not greatly influence our results. Comparisons of the results using Canadian and U.S. data over sub-samples when both are available are also qualitatively similar.

In the next model, we relax the assumption of perfect foresight for the future dividends and the terminal value by using the constant growth or Gordon growth model (GGM). This model assumes that dividends grow at a constant rate into perpetuity, so equity can be valued as follows:

$$E_t[P_t] = E_t[\,D_{t+1}]/\,(r_t - g_t), \qquad (4)$$

where P_t is the price at time t; D_{t+1} is the expected nominal annual dividend paid on the stock at time t+1; r_t is the expected discount rate at time t; and g_t is the expected future annual growth rate of dividends from time t onward. The discount rate is estimated as described above. We estimate the dividend growth rate using several commonly proposed methods. First, we use the arithmetic average of the observed growth rate in dividends with a ten-year rolling window up to and including the dividend paid in year t. Second, we use the arithmetic average of the historic growth rate of dividends using an expanding window with dividends paid from the start of our sample up to and including time t. Our final estimates for the growth rate of dividends are based on the potential relationship between dividend growth and macroeconomic factors. Because macroeconomic factors can influence the profitability of a firm, we estimate the relationship between dividend growth and several key economic factors. The growth rate of GNP is frequently argued to be the maximum sustainable growth rate for a firm's dividends; therefore, we start using an expanding window of the average nominal growth rate in the Canadian economy measured using the GNP since 1871. Other macroeconomic factors also may influence the firms' abilities to pay dividends; therefore, we follow Booth (1998) by forecasting the dividend growth rate using the model:

$$Divgro_t = \alpha_0 + \alpha_1 Yield_t + \alpha_2 Inflation_t + \alpha_3 GNP\ growth_t + \varepsilon_t, \qquad (5)$$

where $Divgro_t$ is the dividend growth rate at time t; $Yield_t$ is the yield on a long-term government bond to represent the opportunity costs for the firm and investors; $Inflation_t$ is the overall rate of inflation based on the year-over-year changes in the consumer price index (CPI) to capture changes in overall level of risk; GNP $Growth_t$ is the year-over-year growth in GNP;[7] and ε_t is the residual error term.

[7] Though Booth (1998) uses GDP, we can only obtain reliable historical estimates for the GNP.

Although the GGM simplifies some of the assumptions from the DDM, it does require assuming that the firm will continue to pay dividends in the future and that the growth rate of dividends will be constant. This latter assumption is non-trivial as it requires the growth rate be lower than the growth rate for the economy over the long term (thus our inclusion of the GNP growth rate in our dividend forecasts).

Relaxing our focus on dividends, we also consider valuation methods based on the firm's earnings. Specifically, we focus on the well known Fed Model. Although the Federal Reserve (i.e., the Fed) does not officially endorse this model, in the 1980s and 1990s it was believed that Fed chairman Alan Greenspan supported the argument that falling interest rates justified higher equity values; this notion subsequently became known as the Fed Model. Formally, this model hypothesizes that the yield on ten-year Treasury bonds should be similar to the earnings yield of the S&P 500 firms (the inverse of the price-earnings ratio), so differences in these yields identify over- or under-priced securities markets. If, for example, the earnings yield on equity is greater than the yield on ten-year Treasury bonds, equity must be under-valued and equity prices should rise. The Fed Model gained popularity because equity prices were seen to decrease (increase) as bond yields increased (decreased) in the 1980s and 1990s.

Though the Fed Model is easy to use, it is frequently criticized for omitting too many important factors. First, it assumes that the earnings yield for firms is a function of only the ten-year bond yield; thus, the model ignores important considerations such as earnings growth or the sustainability of current earnings. It also fails to take into account the impact of possible changes in the equity risk premium. Since these factors are at the heart of the dividend-based valuation models, our study provides a valuable opportunity to investigate the importance of some of these factors on the performance of various valuation techniques.

3. Data

For Bank of Montreal we have collected data on the stock price, dividends and earnings from the Financial Post Investor Suite's Historical Corporate Reports and its predecessor, the hard-copy Financial Post Cards, from 1885 to 2003. For our price series, due to the limited availability of data in the early part of our sample, the annual share price we use is an average of the annual high and low prices. (This is how prices were reported in our data sources.) Dividend and earnings data are available from the Financial Post sources, but wherever possible, we rely on the actual annual reports (with a few gaps) from 1904. These data sources are supplemented by the report produced for the Bank's centenary (Bank of Montreal (1917)) and copies of the *Globe* and *Montreal Gazette* newspapers extending back to the mid-1800s.

For our analysis, determining dividend payments is relatively straightforward. Dividends, as a cash item, are easy to measure and are reported consistently throughout our sample. To understand the role of earnings in dividends we need to determine the size of Bank of Montreal's earnings over time. This latter task has proven slightly more complicated due to the fact that the notion of earnings has changed considerably over the past century. For example, prior to 1916, Bank of Montreal (like other firms) did not pay any taxes. During and shortly after World War I, a "War Tax" was imposed based on bank note circulation. In 1923, a corporate federal income tax (to the "Dominion Government") was imposed. In 1934, provincial taxes were also imposed. We have defined earnings on an after-tax basis over our sample. Another issue worth noting relates to depreciation. Bank of Montreal's 1913 annual report contains the first mention of depreciation—initially referred to as "provisions for bank premises." It appears that depreciation—in the early years at least—was very much a discretionary accounting item. Earnings are determined after depreciation. Beginning in 1973, in accordance with changes in regulations, Bank of Montreal started explicitly reporting an appropriation for loan losses. We estimate earnings after such appropriations.[8] We have adjusted the stock price and dividend data for a 10-for-1 split in 1944; a 5-for-1 split in 1967; and 2-for-1 splits in 1993 and 2001.

For the economic data, we consider publicly available data when possible. The short-term and long-term Treasury bond yields are from the Global Financial Database;[9] U.S. stock market returns are

[8] There were two outlier years related to earnings. In 1987, Bank of Montreal reported net earnings of $550 million before "special provisions for losses on transborder claims" of $765 million, so we use the "before special provisions earnings" (in every other year in our sample the bank reported profits rather than losses). In 1989, we use "net income before country risk provision" of $441 million; increased loan loss provisions in that year reduced the earnings after tax to $51 million.

[9] See http://www.globalfindata.com.

from Professor Robert Shiller's Web site;[10] and the Canadian GNP[11] is supplemented with information from Statistics Canada to obtain data for 2003.

4. Results

4.1 Exploratory Analysis

We begin by characterizing the dividend history and price and earnings histories for Bank of Montreal from its founding in 1817 to 2003 in Tables 1a and 1b. The series have two different fiscal year-ends (April 30 prior to 1904 and October 31 after that date). The total dividends include the regular and special dividends and are presented both unadjusted and adjusted for stock splits. The price and earnings data presented are only those after adjusting for stock splits. Over this period, the bank only had two years during which it did not pay dividends: 1827 and 1828. The increased variability in dividend payments before World War II documented in other studies is clearly evident in our data as well. Between 1817 and 1944, there were many dividend changes (both up and down) generally precipitated by changes in bank earnings. Since World War II, however, the only dividend decline was in 1952, but that was because the special dividend was not paid; there was no change in the regular dividend. This suggests, as discussed in Lintner (1956) and Brav et al. (2005), that managers are hesitant to decrease regular dividend payments—especially more recently.

An interesting aspect of Table 1a is the frequent use of "special" dividends by the bank until 1971 (DeAngelo, DeAngelo and Skinner (2000) also find that special dividends virtually ceased in the U.S. in the early 1970s). Special dividends are useful because they can be used when a firm's ability to continue a certain level of dividend payments is unclear. The 1880s provide a useful example. There was a depression until 1879; conditions improved through 1883, when the real estate bubble in the West burst, resulting in the failure of several banks, followed by a return to prosperity by 1885. Consequently, the bank used special dividends to share profits with its shareholders but did not commit to continuing

---

[10] See http://aida.econ.yale.edu/~shiller/data.htm.
[11] Data obtained from http://library.queensu.ca/webdoc/ssdc/cdbksnew/HistoricalMacroEconomicData/.

payments at this level. Special dividends were frequently used as a precursor to an increase in the regular dividend, especially in the period following World War II.

The earnings in Table 1b are relatively stable until about 1930, decrease until the end of World War II and consistently increase from then until the end of the sample. Similarly, prices remain relatively stable until about 1920, increase until 1929, fall throughout the Depression and World War II and rise rapidly after that time. These tables demonstrate the close relationships between dividends, earnings and prices over time. Since the end of World War II, the growth rate in earnings has been larger and more variable than that of dividends, yet dividends have been increasing at a very stable rate.

Table 2 presents the means and standard deviations for many of the most important changes in firm-level characteristics in the upper panel, whereas the lower panel contains information on macroeconomic factors. To see how these values change over time, we consider the longest period for which we have our dividend series (1818-2003) as well as the full period over which we have all of the data (1885-2003). We next consider each of our three sub-periods: 1885-1913; 1914-1945; and 1946-2003. These sub-periods correspond to pre-World War I, the between-wars period, and post-World War II. The final portion of the table presents p-values for tests of equality of the values across adjacent sub-periods (e.g., the values in 1885-1913 being equal to those in the sub-period from 1914-1945).

Examining the mean annual changes in dividends for Bank of Montreal, we see that since 1818 the annual growth rate has averaged about 2.55%. We find only a small growth in dividends between 1885 and 1913; there is actually a negative growth rate from 1914 to 1945, but a much higher growth in dividends between 1946 and 2003. Earnings generally grew at a slightly faster rate than dividends with a much higher standard deviation. Consistent with the data in Table 1, earnings and dividends appear to move together, with changes in earnings being more volatile and leading dividends. This is consistent with dividends being paid as a relatively stable portion of earnings and managers being reluctant to change dividends in response to changes in earnings; in other words, there exists a general stickiness in dividend policy. There appears to be a break in the link between earnings and dividends following World War II. Before World War II, changes in dividends were more common in both directions and related to changes in earnings; whereas after World War II, dividend policy appeared to become increasingly smooth and conservative.

Since the growth rate in dividends is believed to be limited by the growth rate of the economy, we consider how the changes in dividends relate to changes in the growth rate of the economy. In Table 2, nominal GNP can be seen to grow by 6.7% over the entire 1881-2003 period, and 5.5%; 4.5%; and 8.4% in each of our sub-periods, respectively. The general patterns for GNP and dividend changes are therefore similar, suggesting Bank of Montreal's dividends were, in fact, bounded by the growth in the economy.

Table 2 also demonstrates that, since 1885, over half of Bank of Montreal's total return is attributable to the dividend yield. Because Bank of Montreal's share price shows little change in the earlier periods, almost all the return to shareholders over these periods comes from dividends. As such, Bank of Montreal shares perform much like a bond over this period. In the more recent 1946-2003 period, capital gains have started to play a much larger role in the overall returns for Bank of Montreal (representing about 63% of total returns). These results suggest that either investors have started to value capital gains more than dividends, or that firms are now more actively pursuing growth strategies and are therefore re-investing more funds as opposed to paying them to shareholders in the form of dividends.

Even though the dividend yield appears stable over time, the price-earnings (P/E) ratio illustrates that the earnings yield (the inverse of the P/E ratio) has increased over time (from 5% in the first sub-period to over 7% in the last sub-period). This increase in earnings yield is consistent with the changes in the yields on the long-term bonds, which changes have also increased over this period (going from 3.2% to almost 6%). Consistent with the Fed Model, this observation suggests a relationship between firm earnings and bond yields. Note that the market-to-book ratios are similar in the first and last periods; therefore, this relationship is not the result of changes in how investors value Bank of Montreal's assets.

This analysis suggests that both dividend and earnings-based models may provide valuable insight into the question of how equities should be priced. As a result, we investigate the level of performance and reliability of both the dividend discount model and the Fed Model at describing prices of equity over time.

4.2 Discounted Cash Flow Valuations

Since financial theory suggests that the fundamental price of equity should depend on the discounted value of future dividends. We investigate the ability of the dividend discount model (DDM) in equation (2) and the Gordon growth model (GGM) in equation (4) to explain the actual price of Bank of Montreal shares at each point in time. We also consider the sensitivity of the expected prices obtained using each model with regard to some of the underlying assumptions necessary to implement them.

4.2.1 Estimating the Inputs

Before using these models, we need to estimate the cost of equity and the dividend growth rate. We estimate the discount rate using the CAPM in equation (3), since this is the method most often used by investors. The market premium for this model is estimated using either a moving average of the returns on the market index over the past 30 years[12] or an average using an expanding window starting with the beginning of our equity data in 1871. Though all of the historical information used to estimate the cost of equity was theoretically available to investors, the data may not have been readily available at each point in time. Nevertheless, we assume that investors perform estimations in this fashion (consistent with modern asset pricing theory).

To understand how the estimated or expected costs of equity compare to the costs of equity which appear to have been used by investors to price Bank of Montreal shares, we compare the *expected* costs of equity to the *implied* costs of equity derived from the DDM and the GGM. The implied cost of equity for the DDM is obtained as the discount rate equating the current price with the discounted future value of the actual dividends from each time t until the end of our sample plus the discounted value of the terminal price (the "liquidating dividend") in equation (2). The implied cost of equity using the GGM is obtained as the discount rate equating the actual share price to the expected price using equation (4) with the actual

[12] Note that at the beginning of the sample period, we do not have 30 years of historical data; therefore, we use an expanding window over that period. Once we have 30 years of past market returns, we employ the 30-year moving average window for the rest of our sample.

growth rate of Bank of Montreal dividends from time t until the end of our sample. One interpretation of the implied discount rate is: the rate investors used to value Bank of Montreal shares under the assumption that investors have had rational expectations and could have accurately predicted the future dividends. Since investors are most concerned with the return on equity relative to the return on bonds, we also consider the equity risk premium (the cost of equity less the yield on short-term government bonds).

In Figure 1, we see that the implied costs of equity obtained from both the DDM and GGM follow similar patterns. The main difference is that the cost of equity implied by the DDM is lower than that for the GGM before World War II; they are similar until 1970; and the reverse is true afterward. Comparing the implied costs to the expected costs of equity, we see that the estimated cost of equity using the moving average (MA30) tends to be higher than that using the expanding window (ExpWin)—likely due to the lower expected market risk premium early in the sample. In general, the implied cost of equity using the DDM appears to more closely follow the expected costs of equity. Investigating the differences in the implied and estimated equity risk premia (the costs of equity less the yield on a short-term government bond), we observe a surge in both types of risk premia from the start of the Depression until the end of World War II; however, they then return to the average level by the 1960s. Although Modigliani and Cohn (1979) suggest that investors incorrectly account for inflation, we do not see the differences in the implied and expected costs of equity or equity risk premia being significantly impacted by macroeconomic factors such as inflation.

We test the differences between the implied costs and the estimated costs of equity across various sub-periods in Table 3.[13] The changes over time highlight some of the issues one faces when estimating the cost of equity with historical data. The moving average-based estimates more rapidly respond to changes in market conditions than those using the expanding window. The estimated cost of equity using the moving average is not significantly different from the implied cost of equity over the entire sample

[13] To ensure our results are not driven by the final few values in our sample, we run all of the tests stopping ten, 15 and 20 years before the end of the sample. The results are not significantly different using these two (and other) endpoints. Therefore, only the full results are presented.

period. In the first part of the sample, the cost of equity implied by the DDM is significantly greater than the moving average estimated cost of equity. This suggests that investors required a lower return at this time than our asset pricing models indicate. This is consistent with the belief that investors viewed equity as more bond-like in the early part of the sample. Using the expanding window, the estimated cost of equity is less than the implied cost, using either the DDM or GGM, at the end of the sample. This is most likely because the expanding window includes the low-market risk premium from the early part of the sample, and therefore does not adjust to the changing views regarding the risk premium over the latter part of our sample.[14]

As the capital gains portion of the investors' total return starts to increase, we see a corresponding increase in the costs of equity and in the standard deviation of the costs of equity. The moving average method performs better in the latter period (due to the fact that this period allows our estimated cost of capital to capture the increasing importance of capital gains). We also find slight differences in the implied costs of equity using the DDM and GGM techniques: the implied cost of equity using the GGM is significantly higher in the earlier sub-periods. This is possibly due to our use of rational expectations and the higher growth rates in dividends over our entire sample period than one could have expected at that time. Consequently, our GGM cost of equity may overstate the actual required cost of equity of investors at that time. It is also worth noting that the implied costs of equity for Bank of Montreal are similar to those reported in Foerster and Sapp (2005), who examine measures based on the S&P Index.

For the equity risk premium, we find that the expected and implied equity risk premia are similar in the overall period (especially for the implied value using the DDM). Looking at the sub-periods, we find that the estimated equity risk premia using both the moving average and expanding window methods are greater than the DDM implied value, but are similar to the GGM implied values in the first period. Once again, this is likely due to the perception that dividend-paying stocks were relatively low risk and thus the risk premium was lower than conventional theory would suggest at the beginning of the sample.

[14] In results not presented, we average the estimated cost of equity using the expanding window and moving average techniques and find that it outperforms the individual estimates overall and in each of the sub-periods.

For the latter periods, we find that the expected equity risk premium is lower than the implied equity risk premium, especially for the expanding window techniques. This is likely due to the lower risk premium in the earlier periods which would impact the expanding window estimate more than the moving average based methods which are more robust to these changes over time.

The next input we need to estimate is the future growth rate in dividends. We begin by calculating the actual growth rate of dividends paid from each time t until the end of our sample period (called the perfect foresight growth rate). We compare this to the dividend growth rate we could have estimated using information available at time t. We start with the moving average of the past ten years' growth in dividends and the average for an expanding window from the beginning of the sample to time t. In both cases, we set the growth rate to zero if the average is negative. Since dividends are impacted by macroeconomic forces, we supplement these estimates with macroeconomic-based estimates. The first is based on the economy's growth rate using an expanding window average of the past year-over-year growth rate in the nominal GNP from 1871 (the start of our GNP data) to time t.[15] The second uses a combination of several of the most important economic factors: the economy's growth rate (change in GNP); the inflation rate; and the yield on long-term government bonds (e.g., Booth (1998)).

The estimated and perfect foresight growth rates are compared in Figure 2. The actual growth rate in dividends over our sample period can be seen to be substantially larger than investors would have rationally anticipated. The differences decrease after World War II as dividend policy moved toward a more consistent rate of dividend growth with less focus on a consistent dividend payout ratio. The stable increases in dividends following World War II appear to drive the perfect foresight growth rate over the entire period, making it difficult for our models to forecast dividend growth accurately over both the pre- and post-World War II periods. The estimated or expected growth rate in dividends is low in the early part of the sample because Bank of Montreal changed their dividends as their earnings changed. The frequent increases and decreases in dividends leave it more difficult to predict future dividend changes. Given this information, investors may not have been able to predict the large and persistent increases in

[15] The results are similar using various moving average-based techniques, so they are not presented.

actual dividends over our entire sample period (see Arnott and Bernstein (2002) for an interesting discussion).

The growth rates estimated using macroeconomic factors perform much better than the other estimation techniques. In Figure 2, we compare the performance of the estimated dividend growth rate obtained using the changes in GNP to those using equation (5). The change in GNP performs much better over the entire period. Equation (5) appears unable to capture the changes in how dividends are determined across all sub-periods. It performs reasonably well early and late in the sample.[16] The gradual increase in the GNP, on the other hand, performs reasonably well across all sub-periods.

We formally test the differences between the estimated and perfect foresight dividend growth rates in Table 4. As noted earlier, we see that dividends have grown at a much larger rate over the latter part of the sample. Since it would have been difficult for investors to have predicted this rapid increase in the growth rate of dividends, it is not surprising that—using past dividend growth rates—none of our estimation techniques perform well at predicting the actual growth rate of dividends. This is consistent with studies such as Lintner (1956), who finds that dividend payments were more variable because they were more dependent on earnings and thus firm performance in the early part of our sample. Using macroeconomic factors to forecast dividend growth rates, we find that only the growth in GNP performs well over most of the sample. Our model using important macroeconomic factors consistently underestimates the dividend growth rate (see Figure 2). The predictive ability of the growth in GNP appears to improve over time: the dividend and GNP growth rates both increase at similar rates. The macroeconomic model incorporates economic factors which do not appear to impact dividends over our full sample—though they do appear important in certain sub-periods. Consequently, we find that the general guideline of using GNP growth to approximate long-term growth rate in dividends appears to work well at forecasting the dividends for the stock of a mature, dividend-paying firm.

---

[16] Since the change in dividends was smoother than many of the inputs in the macroeconomic model, several different moving averages of the independent variables and dividend growth rate were also used, yielding similar results.

4.2.2 The Dividend Discount Model

Using our different estimated costs of equity, we discount the actual future dividends and the terminal price to get the expected price per share, as in equation (2). This assumes that investors have rational expectations and thus can accurately predict the true future dividend payments. In Figure 3, we compare the calculated expected prices and the actual prices. The DDM appears to do a reasonable job of explaining the prices across the entire period. There are sustained periods of mispricing, but these are neither systematically overpricing nor underpricing. Until roughly 1920, it appears that there is overpricing relative to the expected prices. This is consistent with the market's tendency to view equity as more bond-like and thus less risky (i.e., higher priced) than our models would have predicted at the time.

Looking at the estimated share prices for the different estimated costs of equity, we see slight underpricing using the moving average cost of equity estimate. This is consistent with the cost of equity being systematically below the implied cost of equity (as discussed earlier). For the expanding window estimate of the cost of equity, we find that—despite the potentially controversial assumption of perfect foresight for dividends in the DDM—the expected share price performs well throughout our sample. It is noteworthy that both methods predicted a decline in share price in the late 1920s.

We formally test the differences between the actual and implied prices in Table 5.[17] The performance of our models improves over time, as the ability to forecast future dividends accurately becomes greater. The similarity between the estimated and actual prices provides evidence to suggest that the dividend discount model provides a useful means for valuing Bank of Montreal over our sample period—especially as dividend growth becomes more stable in the post-World War II years. Although the estimated prices are consistently below the actual prices in the first sub-periods, we find that they are slightly higher than the actual prices in the last sub-period. The differences between these values in the earlier sub-periods is likely related to the relatively minor changes in prices during this period and our estimated cost of equity over-stating the actual cost of equity employed by investors. Consequently, as

[17] Although the results in Table 5 are for the level of prices, we also performed a similar analysis for percentage changes in price to remove the effect of differences in levels over time. The results were similar so they are not presented in order to conserve space.

our estimates for the cost of equity improve (i.e., as equity starts to be viewed as a riskier asset and thus the estimated discount rate works better), so does the predictive ability of the DDM.

4.2.3 Constant Growth Model

To address concerns regarding the ability of investors to forecast dividends over the entire period, we consider the constant growth or Gordon growth model (GGM). This model requires us to forecast only the average future growth rate in dividends. To determine the expected price, we use our estimated growth rates and our estimates for the cost of equity in equation (4). We compare the resulting prices to the actual prices in Figure 4. Until roughly 1910, the expected and actual prices are very similar; the actual price is below the expected price until 1927; the actual price is greater than the expected from the end of World War II until roughly 1970; and both are similar from then until the end of the sample. Many of the differences in prices can be related to the difficulty in using past information to predict the future growth rate of dividends accurately: investors over-estimated the increases in dividends around World War I and under-estimated them for the first period after the end of World War II.

Once again, the expected share price obtained using the moving average to estimate the cost of equity is less than the actual price except around World War I; the early 1920s; and the 1980s—the periods when the GNP growth rate would have over-estimated the Bank of Montreal dividend growth rate (see Figure 2). This is confirmed by the expected price obtained using the macroeconomic model which more significantly under-estimates the growth rate to be more significantly below the actual price. The expected price using the expanding window cost of equity follows a similar pattern but appears to perform better overall. As before, these models suggest that the price should decrease in the late 1920s, thus appearing to predict the subsequent market crash. We do not, however, find a systematic relationship between these pricing errors and any other macroeconomic factors (such as inflation).

We test the differences between the actual and implied prices in Table 6.[18] As in Table 5, we find that the performance of the GGM differs over time. However, unlike the results for the DDM, the estimated fundamental value for the GGM appears to overstate the prices in the early periods[19] but is much closer to the actual prices otherwise. Consequently, we cannot reject the null hypothesis that the fundamental prices and the actual prices are the same for most of the cases we consider when we use the GNP growth rate to forecast future growth.

4.2.4 Earnings-Based Model

Building on the apparent relationships between earnings and the price of Bank of Montreal shares documented earlier, we consider the Fed Model. Although the Fed Model does not make specific predictions regarding the expected price of equity, it does make predictions regarding the fair value of the earnings yield relative to the ten-year government bond yield. In Figure 5, we see that the earnings yield for Bank of Montreal is approximately 2% greater than the bond yield from the beginning of the sample until about 1958. Between 1958 and 1970 both are almost the same, but since 1970, the earnings yield is much greater than the bond yield. According to the Fed Model, these results suggest that Bank of Montreal's stock has been under-valued except during the 1960s.

These results suggest that there is more to valuing a firm than simply the comparison of the earnings yield to the current bond yield. Comparing the predictions from the Fed Model to our comparisons using the DDM and GGM, we do not find many apparent relationships because the Fed Model suggests that Bank of Montreal is generally undervalued. Looking carefully at the costs of equity, however, it appears that the periods during which the equity markets perform the best (and the estimated cost of equity is much larger than the implied cost of equity) are when the Fed Model works the best. This potential relationship between strong market performance and the accuracy of the Fed Model's

[18] As with the estimates for the fundamental price from the DDM, we also perform the analysis for Table 6 using percent changes in prices but do not present the results as they are qualitatively similar to those presented for levels.
[19] This is likely the result of the over-estimation of dividend growth using GNP growth at this time (see Figure 2).

predictions would help to explain the value of the Fed Model during the strong equity markets in the 1980s and 1990s.

Despite this apparent failure of the Fed Model for a company such as the Bank of Montreal, we do learn several lessons from this model. First, we find evidence that the earnings yield and the ten-year bond yield are correlated. Although we do not find that Bank of Montreal is priced to equate the earnings yield to the bond yield, we do find that the market values equity more (less) as the government bond yields decrease (increase) as predicted by the Fed Model. Second, the difference between the earnings yield and the bond yield is consistent—suggesting the model may be missing something. Since dividends are paid as a portion of earnings, dividends may be a more consistent means of measuring the return to investors from holding equity for companies with regular dividend payments (such as BMO). Finally, the results highlight the difficulties that our valuation techniques have at valuing stock during volatile periods such as those experienced in North America in the latter part of the twentieth century.

5. Conclusions

Overall, our study provides interesting evidence that confirms the ability of discounted cash flow-based techniques to explain the value of equity for a large, mature, dividend-paying company. We provide a detailed clinical study of the role of dividends and earnings in the valuation of equity. Because discounted cash flow models and models considering firms' earnings are so commonly used in practice, it is important to understand how well they price assets and the sensitivity of the calculated fundamental prices to the assumptions used when implementing these models. Although these issues are frequently raised in textbooks, they are rarely investigated and discussed.

Even though the majority of the total return received by investors over the past century has come in the form of dividends (especially in the earlier part of the sample), recent studies suggest a decrease in their importance as the impact of capital gains on investors' total returns has increased. Despite this increase in the role for capital gains, we find that the observed changes in the market price for Bank of Montreal continue to be highly correlated with changes in dividends. This suggests an important role for fundamental valuation methods. We find that the dividend discount model (DDM) and Gordon growth or constant growth model (GGM) both perform well at explaining the observed price for one firm that has a long history of paying dividends under some of the most commonly used assumptions. However, earnings-based models—such as the Fed Model—do not perform as well. Thus, our analysis suggests that dividend-based discounted cash flow models appear to be reasonable models applied at the firm level for a company that pays a continuous and growing stream of dividends.

Specifically, the DDM—using perfect foresight regarding the future dividend payments and using the capital asset pricing model (CAPM) to estimate the discount rate—does a reasonable job of explaining prices at each point in time, especially using the full history of market returns in the CAPM. Since perfect foresight of dividends is unlikely, we investigate the performance of the GGM, which requires one to forecast only the future growth rate of dividends. Using standard methods for estimating the growth rate of dividends, the model obtains reasonable prices. We do, however, find differences in the characteristics of the expected prices obtained using each of the various ways to predict dividend growth. Interestingly, the growth rate in the economy as measured by the GNP is one of the best performing estimates. Other techniques, based on the historical growth rate in dividends or a wider set of macroeconomic factors obtain much more variable expected prices; thus, their performance is less consistent. Nevertheless, both methods appear to provide accurate estimates for the future value of the asset, while simultaneously highlighting the need for caution in the application of the different techniques.

References

Aliber, R, 1984, “International Banking: A Survey”, *Journal of Money, Credit and Banking*, Vol. 16, No. 4, Part 2: Bank Market Studies. (Nov., 1984), pp. 661-678.

Allen, Franklin, and Roni Michaely, 2002, “Payout Policy” North-Holland Handbook of Economics edited by George Constantinides, Milton Harris, and Rene Stulz; North-Holland.

Arnott, Robert and Peter L. Bernstein, 2002, “What Risk Premium Is "Normal?" *Financial Analyst Journal* Vol 58 (2).

Bagwell, Laurie Simon and John Shoven, 1989, "Cash Distributions to Shareholders," *Journal of Economic Perspectives* Vol. 3(3), pp. 129-40.

Baker, Malcolm, and Jeffrey Wurgler, 2004a, “A catering theory of dividends”, *Journal of Finance* 59, 271-288.

Baker, Malcolm and Jeffrey Wurgler, 2004b, “Appearing and Disappearing Dividends: The Link to Catering Incentives,” *Journal of Financial Economics*, forthcoming.

Bank of Montreal, 1917, “The Centenary of the Bank of Montreal: 1817-1917”, The Herald Press and Advertising Agency.

Booth, L, 1998, "A New Model for Estimating Risk Premiums (along with some evidence of their decline”, *Journal of Applied Corporate Finance* 11, pp. 109-120.

Booth, L, 2001, "Equity Market Risk Premiums in the U.S. and Canada" *Canadian Investment Review* 14-3, pp. 34-43.

Brav, Alon John R. Graham, Campbell R. Harvey, and Roni Michaely, 2005, “Payout policy in the 21st century”, *Journal of Financial Economics*, forthcoming.

Claus, J and J. Thomas, 2001, “Equity premia as low as three percent? Empirical evidence from analysts’earnings forecasts for domestic and international stock markets”, *Journal of Finance* 56(5), pp. 1629-1666.

DeAngelo, Harry, Linda DeAngelo, and Douglas Skinner, 2000, “Special dividends and the evolution of dividend signaling,” *Journal of Financial Economics, 57*, 309 -354.

Fama, E. and K. French, 1988, “Dividend Yields and Expected Stock Returns”, *Journal of Financial Economics* 22, pp. 3-25.

Fama, Eugene F., and Kenneth R. French, 1999,"The Corporate Cost of Capital and the Return on Corporate Investment," *Journal of Finance* Vol. 57, pp. 637-659.

Fama, Eugene, and Kenneth French, 2001, “Disappearing dividends: Changing firm characteristics or lower propensity to pay?”, *Journal of Financial Economics* 60, 3-43.

Fama, Eugene F., and Kenneth R. French. 2002, “The Equity Risk Premium”, *Journal of Finance*, Vol. 57, pp. 637-659.

Foerster, S. and S. Sapp, 2005, “Dividends and Stock Valuation: A Study From the Nineteenth to the Twenty-First Century”, Working Paper, Ivey Business School.

Gordon, M. J., 1962, The Investment, Financing and Valuation of the Corporation, R. Irwin, Homewood Ill.

Gordon, M.J. and E. Shapiro, 1956, “Capital equipment analysis: the required rate of profit”, *Management Science* 3, pp. 102–110.

Goyal, Amit, and Ivo Welch, 2003, "Predicting the Equity Premium with Dividend Ratios," *Management Science* 49(5), pp. 639–654.

Grullon, G., R. Michaely, 2002, “Dividends, Share Repurchases, and the Substitution Hypothesis”, *Journal of Finance* Vol. 57, pp. 1649-1684.

Jagannathan, Ravi, Ellen R. McGrattan and Anna Scherbina, 2000, "The declining U.S. equity premium," Quarterly Review, Federal Reserve Bank of Minneapolis 24 (4), pp. 3-19.

Lintner, J., 1956, "Distribution of Incomes or corporations among dividends, retained earnings and taxes", *American Economic Review* 46, pp. 97-113.

Pastor, Lubos and Robert Stambaugh, 2001, "The Equity Premium and Structural Breaks", *Journal of Finance* 56(4), pp. 1207-39.

Shaffer, S, 1993, "Test of Competition in Canadian Banking", *Journal of Money, Credit and Banking*, Vol. 25, No. 1. (Feb., 1993), pp. 49-61.

Welch, Ivo, 2000, "Views of Financial Economists On The Equity Premium And Other Issues", The *Journal of Business* 73 (4), pp. 501-537.

Williams, John, 1938, The Theory of Investment Value, North Holland Publishing.

Appendix: History of the Bank of Montreal

Since Bank of Montreal has played such an important role in Canadian banking and the Canadian economy, it is useful to understand its history as well as the nature of Canadian banking. In Canada, the largest six banks, including Bank of Montreal, have a national presence and represent the vast majority of market share using almost every banking-related metric (i.e., total assets, total loans, mortgage loans, etc.). The Canadian banking sector is different from its American counterpart in its decentralized nature. Because of their geographical diversification, Canadian banks have been better able to weather economic storms (such as the Great Depression) than banks in the U.S. Despite the increased concentration of the Canadian banking industry, this has not led to a decrease in competition (see Shaffer (1993)). Aliber (1984) surveys the literature on international banking practices and suggests that the U.S. is a unique model and that Canada is more typical of the markets in other countries. It even appears that the U.S. banking industry is moving toward a more concentrated model (such as that in Canada).

The Bank of Montreal is an important firm to consider in its own right. It is the oldest bank in Canada and it was considered one of the major corporations in North America through the early part of our sample; it was even referred to as "the greatest bank in [North] America." The bank originated in 1817 to facilitate trade in Canada. The first year went so well that halfway through the first year, the bank decided to share some of the prosperity with its suppliers of capital by paying a dividend. After paying the dividend, the directors decided to put the remainder of the profits into a reserve fund. This became known as "the rest." The rest became an important feature of Bank of Montreal by allowing it to weather adverse financial conditions and continue to pay regular dividends to investors without having to encroach on the original capital. The only time the bank missed paying a dividend was following the financial collapse in England in 1825. The collapse caused severe losses to merchants in Canada, which losses were felt throughout the region for the next two years. As a consequence, Bank of Montreal had to skip its dividend payments in 1827 and 1828. It was able to return to paying a small dividend in 1829 and has continued to pay a dividend each year from then to the present. In the early years, the changes in the size of dividends can be seen as closely following earnings fluctuations.

As Canada approached Confederation (in 1867), Bank of Montreal had one-quarter of the banking assets in Canada and was the government's depositary and fiscal agent. From this time, the bank continued to grow as Canada expanded though increased immigration from Europe and the expansion west. At the beginning of the twentieth century, the bank had gained a preeminent position:

> "It may be interesting to note that the largest total dividend paid in Canada by one concern is by the Bank of Montreal…which, by the way, after the Bank of England is the second largest banking institution in the world" *The Globe* (January 2, 1901).

Despite its ups and downs, the bank generally maintained its regular dividend with the occasional use of special dividends until the start of the Great Depression in the early 1930s. The Great Depression and the transferring of Bank of Montreal's role as Canada's central bank to the Bank of Canada in 1934 led to declining earnings and Bank of Montreal's first decrease in dividends in years. The dividend did not increase again until after the end of World War II; since that time, it has increased relatively consistently.

Table 1a

Bank of Montreal Dividend History

Dividends paid on each share of BMO from 1817 to 2003 (initial dividend is based on $100 stock price).

Fisc. Year-end	Regular Div.	Special Div.	Total Div.	Adj. Div.	Fisc. Year-end	Regular Div.	Special Div.	Total Div.	Adj. Div.
1818	3.00		3.00	3.00	1942	7.50		7.50	7.50
1819	8.00		8.00	8.00	1943	6.00		6.00	6.00
1820	6.50		6.50	6.50	1944	6.00		6.00	6.00
1821-1825	6.00		6.00	6.00	1945	0.60*		0.60*	6.00
1826	3.00		3.00	3.00	1946	0.80	0.15	0.95	9.50
1827-1828	0.00		0.00	0.00	1947-1949	0.80	0.20	1.00	10.00
1829	2.50		2.50	2.50	1950	1.00		1.00	10.00
1830-1831	6.00		6.00	6.00	1951	1.00	0.20	1.20	12.00
1832	7.00	5.00	12.00	12.00	1952	1.00		1.00	10.00
1833-1835	8.00	6.00	14.00	14.00	1953-1954	1.20	0.20	1.40	14.00
1836	8.00	4.00	12.00	12.00	1955	1.25	0.20	1.45	14.50
1837	8.00	16.00	24.00	24.00	1956-1957	1.40	0.20	1.60	16.00
1838	6.00		6.00	6.00	1958	1.45	0.20	1.65	16.50
1839	7.00		7.00	7.00	1959	1.60	0.20	1.80	18.00
1840-1841	6.00		6.00	6.00	1960	1.65	0.25	1.90	19.00
1842-1843	7.00		7.00	7.00	1961	1.80	0.25	2.05	20.50
1844	6.00		6.00	6.00	1962	1.80	0.30	2.10	21.00
1845	7.00		7.00	7.00	1963	1.90	0.25	2.15	21.50
1846-1847	7.50		7.50	7.50	1964	2.10	0.10	2.20	22.00
1848	7.00		7.00	7.00	1965	2.13	0.15	2.28	22.75
1849-1851	6.00		6.00	6.00	1966	2.25	0.13	2.38	23.75
1852	6.50		6.50	6.50	1967	0.48*	0.04*	0.52*	26.00
1853-1854	7.00		7.00	7.00	1968	0.56	0.02	0.58	29.00
1855	7.50		7.50	7.50	1969	0.67	0.03	0.70	35.00
1856-1857	8.00		8.00	8.00	1970	0.72	0.03	0.75	37.50
1858	7.00		7.00	7.00	1971	0.75	0.02	0.77	38.50
1859-1866	8.00		8.00	8.00	1972	0.84		0.84	42.00
1867-1868	10.00		10.00	10.00	1973	0.90		0.90	45.00
1869	11.00		11.00	11.00	1974-1975	0.96		0.96	48.00
1870	12.00		12.00	12.00	1976	0.98		0.98	49.00
1971-1874	12.00	4.00	16.00	16.00	1977	1.02		1.02	50.75
1875-1876	14.00		14.00	14.00	1978	1.09		1.09	54.50
1877	13.00		13.00	13.00	1979	1.32		1.32	66.00
1878	12.00		12.00	12.00	1980	1.54		1.54	77.00
1879	10.00		10.00	10.00	1981	1.80		1.80	90.00
1880	9.00		9.00	9.00	1982-1986	1.96		1.96	98.00
1881	8.00	2.00	10.00	10.00	1987-1988	2.00		2.00	100.00
1882	9.00	1.00	10.00	10.00	1989-1992	2.12		2.12	106.00
1883-1884	10.00		10.00	10.00	1993	1.12*		1.12*	112.00
1885-1886	10.00	1.00	11.00	11.00	1994	1.20		1.20	120.00
1887	10.00	2.00	12.00	12.00	1995	1.32		1.32	132.00
1888-1911	10.00		10.00	10.00	1996	1.48		1.48	148.00
1912-1918	10.00	2.00	12.00	12.00	1997	1.64		1.64	164.00
1919	12.00		12.00	12.00	1998	1.76		1.76	176.00
1920-1930	12.00	2.00	14.00	14.00	1999	1.88		1.88	188.00
1931	12.00		12.00	12.00	2000	1.00*		1.00*	200.00
1932	11.00		11.00	11.00	2001	1.12		1.12	224.00
1933	8.50		8.50	8.50	2002	1.20		1.20	240.00
1934-1941	8.00		8.00	8.00	2003	1.34		1.34	268.00

* indicates year with stock splits: 1945: 10 for 1, 1967: 5 for 1, 1993: 2 for 1, and 2000: 2 for 1

Table 1b
Bank of Montreal Price and Earnings History

Adjusted earnings and adjusted average price per share for the Bank of Montreal from 1885 to 2003. The values are adjusted for stock splits over this period.

	Adj EPS	Adj price		Adj EPS	Adj Price		Adj EPS	Adj Price
1885	11.61	197.00	1925	14.42	255.50	1965	30.83	649.38
1886	12.22	222.00	1926	14.57	263.50	1966	32.77	580.63
1887	12.67	225.75	1927	14.87	313.75	1967	34.11	617.50
1888	10.70	223.50	1928	15.87	377.00	1968	29.73	681.25
1889	11.48	229.88	1929	15.95	357.50	1969	50.61	778.13
1890	11.48	224.50	1930	14.61	300.00	1970	52.52	778.13
1891	7.04	222.75	1931	12.43	267.50	1971	56.14	818.75
1892	11.05	227.50	1932	11.40	187.50	1972	77.78	1040.63
1893	11.05	221.00	1933	9.43	185.50	1973	83.32	975.00
1894	10.94	223.50	1934	8.62	187.00	1974	65.83	796.88
1895	10.09	220.50	1935	8.35	178.00	1975	118.72	787.50
1896	10.34	217.88	1936	8.28	202.50	1976	100.73	753.13
1897	10.25	233.63	1937	8.36	217.00	1977	114.25	793.75
1898	10.54	238.00	1938	8.05	210.00	1978	150.44	1078.13
1899	11.25	254.50	1939	8.23	199.50	1979	172.78	1215.63
1900	12.70	256.75	1940	8.16	191.50	1980	167.28	1437.50
1901	12.81	256.75	1941	8.16	182.00	1981	229.61	1393.75
1902	13.34	264.38	1942	7.73	156.50	1982	205.46	1096.88
1903	13.55	262.75	1943	7.79	153.50	1983	216.07	1490.63
1904	11.49	249.19	1944	7.48	163.75	1984	201.00	1265.63
1905	11.38	255.50	1945	8.15	191.25	1985	219.04	1500.00
1906	12.49	256.63	1946	12.47	244.38	1986	217.16	1553.13
1907	13.75	240.88	1947	15.06	267.50	1987	269.28	1584.38
1908	13.22	239.00	1948	15.17	265.00	1988	259.15	1359.38
1909	12.68	249.75	1949	16.16	266.25	1989	199.12	1587.50
1910	12.49	250.75	1950	16.51	288.75	1990	227.13	1353.13
1911	15.29	253.00	1951	14.88	287.50	1991	249.19	1896.88
1912	15.74	248.50	1952	15.75	294.35	1992	261.42	2265.63
1913	13.52	235.50	1953	19.56	341.25	1993	284.63	2253.13
1914	13.79	238.75	1954	16.72	419.35	1994	310.79	2693.75
1915	13.68	237.00	1955	17.87	478.75	1995	373.93	2831.25
1916	12.75	227.00	1956	19.95	538.75	1996	449.34	3777.50
1917	13.86	222.00	1957	18.02	455.00	1997	499.17	5497.50
1918	17.60	213.00	1958	19.18	481.25	1998	510.53	6842.50
1919	16.58	216.00	1959	20.13	575.63	1999	517.54	5947.50
1920	15.45	202.75	1960	23.42	546.88	2000	710.70	5707.00
1921	15.14	206.00	1961	24.00	681.25	2001	601.53	7581.00
1922	14.14	223.50	1962	26.36	626.88	2002	575.43	7165.00
1923	14.37	237.00	1963	27.57	657.50	2003	734.54	8805.00
1924	14.37	240.00	1964	29.39	649.38			

Table 2
Summary Statistics

Means (and standard deviations below) for annual dividends, earnings, returns, and valuation measures for the Bank of Montreal in the top panel. Annual dividend, earnings, and price changes are based on logarithmic changes. The tests for differences in the means are p-values for the t-tests. The tests for differences in the standard deviations are the p-values from F-tests. Period 1 is 1885-1913, period 2 is 1914-1945, and period 3 is 1946-2003. The lower panel presents means (and standard deviations below) for the macroeconomic factors: nominal GNP growth, inflation and long-term and short-term bond yields.

	Full Period		Sub-Periods			Tests (p-values)	
	1818-2003	1885-2003	1885-1913	1914-1945	1946-2003	1 vs 2	2 vs 3
change in dividends	2.55%	2.76%	0.63%	-2.17%	6.55%	0.09	0.00
	18.85%	*8.54%*	*5.41%*	*7.28%*	*8.79%*	*0.12*	*0.26*
change in earnings		3.46%	0.43%	-1.58%	7.76%	0.52	0.00
		14.76%	*14.90%*	*7.62%*	*16.52%*	*0.00*	*0.00*
dividend payout		73.65%	87.40%	90.84%	57.29%	0.25	0.00
		22.39%	*13.71%*	*8.54%*	*19.27%*	*0.01*	*0.00*
dividend yield		4.59%	4.34%	4.83%	4.59%	0.01	0.37
		1.18%	*0.46%*	*0.95%*	*1.49%*	*0.00*	*0.01*
capital gain		3.22%	0.64%	-0.65%	6.60%	0.52	0.01
		12.16%	*3.85%*	*10.52%*	*14.60%*	*0.00*	*0.05*
total return		7.78%	4.93%	4.16%	11.15%	0.70	0.02
		11.85%	*3.86%*	*10.33%*	*14.16%*	*0.00*	*0.06*
P/E ratio		17.09	20.21	19.54	14.18	0.44	0.00
		6.22	*2.89*	*3.86*	*7.14*	*0.13*	*0.00*
Market-to-book ratio		1.32	1.44	1.07	1.40	0.00	0.00
		0.32	*0.14*	*0.26*	*0.35*	*0.00*	*0.07*
GNP growth rate		6.68%	5.48%	4.47%	8.35%	0.32	0.00
		3.62%	*2.90%*	*4.31%*	*2.43%*	*0.08*	*0.00*
Inflation		2.80%	0.73%	1.64%	4.16%	0.22	0.00
		2.83%	*1.90%*	*3.32%*	*1.97%*	*0.02*	*0.00*
Long-term government bond		4.59%	3.24%	3.28%	5.99%	0.80	0.00
		2.46%	*0.21%*	*0.91%*	*2.85%*	*0.00*	*0.00*
Short-term government bond		4.67%	5.04%	3.11%	5.35%	0.00	0.00
		2.64%	*1.01%*	*2.16%*	*3.08%*	*0.00*	*0.00*

Table 3: Expected and Implied Costs of Equity

Comparison of two measures of the expected cost of equity and two measures of the implied cost of equity for Bank of Montreal. Expected measures are based on the Capital Asset Pricing Model (CAPM) in equation (3). The risk-free rate is the yield on the short-term government bond each year and the Bank of Montreal beta is assumed to be 1.0. The first expected cost of equity estimates the market risk premium using a moving average of equity versus government bond returns over the last 30 years (MA) while the expanding window technique uses data from the start of the sample to each particular year (ExpWin). These values are compared to the implied values obtained using the Dividend Discount Model (DDM) in equation (2) and the Gordon Growth Model (GGM) in equation (4). The DDM uses actual dividends and a terminal value, while the GGM uses the actual growth rate of dividends from each particular year until the end of the sample. The expected and implied equity risk premia are calculated using the expected and implied costs of equity less the yield on the short-term government bond in each year. Panel A presents averages and standard deviations over various sub-periods, while Panel B presents the t-statistics from difference of means tests.

Panel A: Averages and standard deviations

	Expected Cost of Equity		Implied Cost of Equity		Equity Risk Premium		Implied Equity Risk Premium	
	MA	ExpWin	GGM	DDM	MA	ExpWin	GGM	DDM
Averages								
1885-2002	10.22%	8.90%	10.63%	10.27%	5.52%	4.20%	5.83%	5.57%
1885-1992	9.98%	8.73%	9.64%	9.04%	5.28%	4.03%	4.94%	4.33%
1885-1917	7.48%	7.48%	7.63%	5.86%	2.49%	2.49%	2.62%	0.87%
1918-1944	9.19%	8.31%	9.33%	7.91%	6.21%	5.33%	6.35%	4.93%
1945-2002	12.27%	9.99%	12.99%	13.88%	6.93%	4.65%	7.42%	8.54%
1945-1992	12.15%	9.83%	11.21%	11.85%	6.68%	4.35%	5.73%	6.38%
Standard Deviations								
1885-2002	2.41%	1.27%	8.63%	6.60%	3.36%	2.67%	8.94%	6.89%
1885-1992	2.35%	1.19%	1.88%	3.25%	3.37%	2.70%	2.75%	3.52%
1885-1917	0.82%	0.71%	0.56%	0.42%	0.91%	0.81%	1.19%	1.07%
1918-1944	1.37%	0.63%	0.77%	1.36%	2.77%	2.52%	2.45%	3.37%
1945-2002	1.26%	0.56%	11.91%	7.83%	3.42%	2.99%	12.32%	8.35%
1945-1992	1.23%	0.46%	1.45%	2.65%	3.61%	3.16%	2.72%	2.88%

Panel B: Estimated t-statistics for differences in means test

	MA vs GGM	MA vs DDM	ExpWin vs GGM	ExpWin vs DDM
Cost of Equity				
1885-2002	-0.49	-0.07	-2.15	-2.20
1885-1992	1.16	2.44	-4.24	-0.91
1885-1917	-0.86	9.94	-0.93	11.08
1918-1944	-0.46	3.37	-5.23	1.36
1945-2002	-0.45	-1.53	-1.90	-3.75
1945-1992	3.39	0.69	-6.19	-5.16

	MA vs GGM	MA vs DDM	ExpWin vs GGM	ExpWin vs DDM
Equity Risk Premium				
1885-2002	-0.24	0.08	-1.81	-1.90
1885-1992	1.43	2.65	-2.13	-0.30
1885-1917	6.08	13.85	6.38	14.61
1918-1944	-0.59	1.38	-2.24	0.25
1945-2002	-0.68	-1.90	-2.05	-3.90
1945-1992	0.51	-0.49	-3.43	-4.42

Table 4: Estimated Dividend Growth Rates

Comparison of the estimated growth rates in dividends for Bank of Montreal used in the Gordon Growth Model or GGM in equation (4). The first column of data (Perfect Foresight) presents the actual growth rate in dividends from each year until the end of our sample. The next three columns present the actual past growth rate in dividends (Actual), as well as the moving average of the past 10 years (MA) and the expanding window of past dividend growth rates (ExpWin) (Note: both have a minimum value of zero). The fourth column (GNP) presents the nominal growth rate in GNP from the start of our GNP sample data in 1871 to a particular year as an estimate of expected dividend growth. The final column (Macroeconomic Model) represents the predictions for dividend growth using the macroeconomic model in equation (5). Panel A presents averages and standard deviations over various sub-periods, while Panel B presents the t-statistics from various differences of means tests.

Panel A: Averages and standard deviations

	Perfect Foresight	Actual	MA	ExpWin	GNP	Macroeconomic Model
Averages						
1885-2002	5.12%	2.61%	3.53%	0.83%	4.61%	2.30%
1885-1992	4.88%	2.11%	2.87%	0.65%	4.47%	2.27%
1885-1917	3.17%	0.27%	0.51%	0.19%	3.13%	0.67%
1918-1944	4.62%	-2.57%	0.80%	0.26%	4.32%	0.56%
1945-2002	6.47%	6.30%	6.53%	1.46%	5.57%	4.02%
1945-1992	6.20%	5.96%	5.65%	1.20%	5.48%	4.30%
Standard Deviations						
1885-2002	1.70%	8.52%	6.03%	1.04%	1.23%	2.80%
1885-1992	1.46%	8.64%	3.07%	0.85%	1.20%	2.91%
1885-1917	0.28%	5.23%	1.05%	0.77%	0.93%	1.57%
1918-1944	0.97%	7.85%	1.06%	0.28%	0.48%	2.84%
1945-2002	1.15%	8.65%	7.46%	1.02%	0.53%	2.24%
1945-1992	0.66%	9.29%	2.43%	0.80%	0.53%	2.32%

Panel B: Estimated t-statistics for differences in means tests

	Perfect Foresight vs MA	Perfect Foresight vs ExpWin	Perfect Foresight vs GNP	Perfect Foresight vs Macroeconomic model
1885-2002	2.73	23.22	-2.63	9.28
1885-1992	6.12	25.89	-2.22	8.29
1885-1917	13.85	20.54	-0.23	8.89
1918-1944	13.58	22.00	-1.42	6.91
1945-2002	-0.05	24.61	-5.38	7.37
1945-1992	1.50	33.11	-5.83	5.40

Table 5: Estimated Fundamental Price Based on the Dividend Discount Model (DDM)

Comparison of the actual and fundamental (or estimated) share price for Bank of Montreal. The first column of data contains the actual Bank of Montreal price (Actual). The second column of data contains the estimated fundamental price based on the DDM in equation (2) and the cost of equity estimated using the 30-year moving average technique to estimate the market risk premium based on CAPM (MA). The final column contains the estimated fundamental price based on the DDM and the cost of equity estimated using the expanding window technique to estimate the market risk premium based on CAPM (ExpWin). Panel A presents averages and standard deviations over various sub-periods, while Panel B presents the t-statistics from various differences of means tests.

Panel A: Averages and standard deviations

	Actual	MA	ExpWin
Averages			
1885-2002	835.30	800.88	946.44
1885-1992	505.54	502.83	626.34
1885-1917	237.15	164.56	162.98
1918-1944	225.45	164.03	195.71
1945-2002	1470.47	1470.94	1755.62
1945-1992	847.62	925.96	1187.13
Standard Deviations			
1885-2002	1328.29	1247.46	1360.28
1885-1992	449.53	684.92	776.01
1885-1917	15.99	26.88	22.98
1918-1944	56.98	44.41	39.04
1945-2002	1688.87	1526.75	1591.21
1945-1992	492.88	858.65	889.54

Panel B: Estimated t-statistics for the differences in means tests

	Actual vs MA	Actual vs ExpWin
1885-2002	-0.63	0.28
1885-1992	-1.39	-0.58
1885-1917	14.99	17.43
1918-1944	2.20	2.34
1945-2002	-0.93	0.11
1945-1992	-2.29	-1.41

Table 6: Estimated Fundamental Price Based on the Gordon Growth Model (GGM)

Comparison of the actual and fundamental (or estimated) share price for Bank of Montreal. The first column of data contains the actual Bank of Montreal price (Actual). The second column of data contains the estimated fundamental price based on the GGM in equation (4) and the cost of equity estimated using the 30-year moving average technique to estimate the market risk premium based on CAPM and GNP growth rates. The final column contains the estimated fundamental price based on the GGM and the cost of equity estimated using the expanding window technique to estimate the market risk premium based on CAPM and GNP growth rates. Panel A presents averages and standard deviations over various sub-periods, while Panel B presents the t-statistics for various differences of means tests.

Panel A: Averages and standard deviations

	Actual Price	Price (ke MA30,g GNP)	Price (ke ExpWin, g GNP)
Averages			
1885-2002	835.30	1405.55	2137.63
1885-1992	505.54	504.99	655.94
1885-1917	237.15	256.53	250.91
1918-1944	225.45	249.81	290.25
1945-2002	1470.47	2618.23	4105.03
1945-1992	847.62	819.36	1140.11
Standard Deviations			
1885-2002	1328.29	8219.26	13867.03
1885-1992	449.53	580.69	753.90
1885-1917	15.99	80.54	61.72
1918-1944	56.98	143.33	140.63
1945-2002	1688.87	11704.94	19764.36
1945-1992	492.88	755.12	921.40

Panel B: Estimated t-statistics for the differences in means tests

	Actual vs MA	Actual vs ExpWin
1885-2002	-1.01	-0.19
1885-1992	-1.77	-0.39
1885-1917	-1.22	-8.72
1918-1944	-2.18	-3.18
1945-2002	-1.00	0.09
1945-1992	-1.92	1.94

Figure 1
Estimated and Implied Costs of Equity for Bank of Montreal, 1885-2003

Comparison of the estimated and implied costs of equity for Bank of Montreal. We estimate the cost of equity using both a 30 year moving average (MA30) and an expanding window (ExpWin) to estimate the market premium to be used in the CAPM to estimate the cost of equity. The implied costs of equity are calculated using the DDM and GGM and the actual dividends paid from time t to the end of the sample.

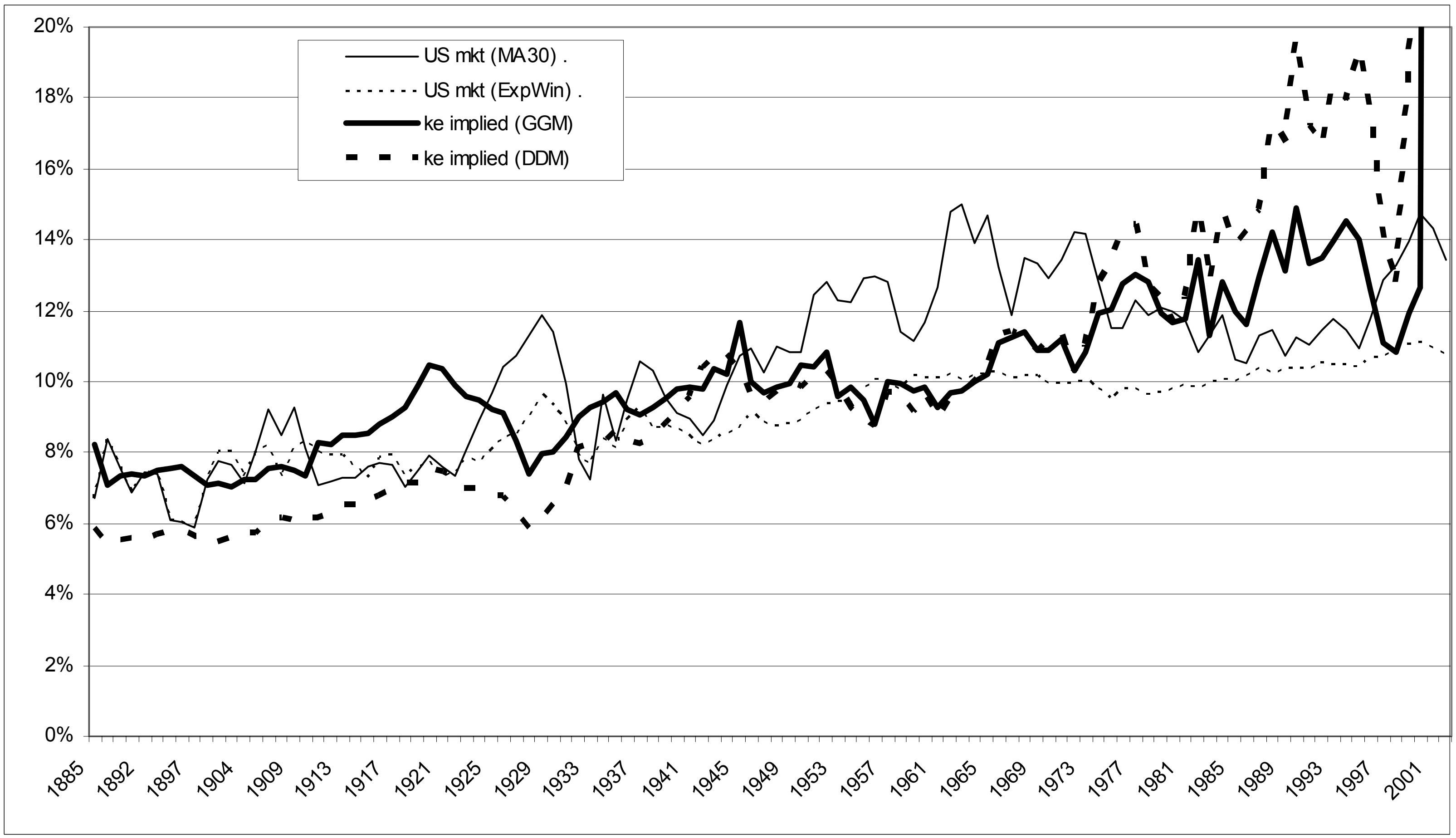

Figure 2
Estimated Dividend Growth Rates for Bank of Montreal, 1885-2003

Comparison of the estimated growth rates in dividends for Bank of Montreal. The perfect foresight growth rate is actual nominal growth rate in dividends from each year until the end of our sample. We also present the 10-year moving average of Bank of Montreal dividend growth, with a minimum value of zero, up to a particular year (g MA(10)), the cumulative average of Bank of Montreal dividend growth, with a minimum value of zero, from 1871 up to a particular year (g ExpWin)). Using macroeconomic factors we also present the nominal growth rate in GNP from the start of our GNP sample data in 1871 to a particular year (GNP MA(10)), and estimates using a model with several macroeconomic factors described in equation 5 (Macro Model).

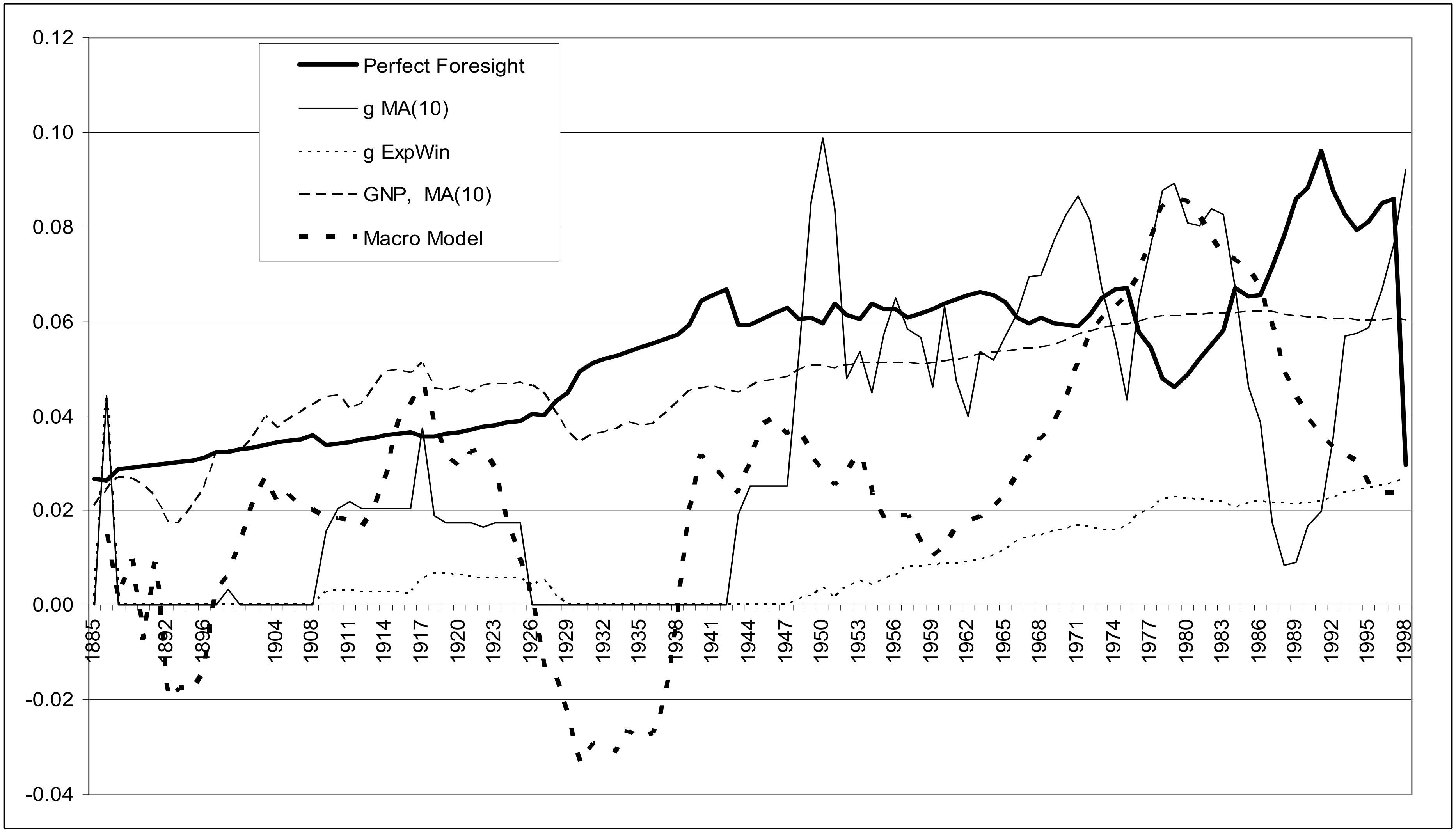

Figure 3

Actual and Estimated Fundamental Bank of Montreal Prices Based on the Dividend Discount Model (DDM), 1885-2003

Comparison of the actual and fundamental or estimated share price for Bank of Montreal. We include the actual Bank of Montreal price (Actual Price) and the estimated fundamental price based on the DDM in equation (2) with the cost of equity estimated using the 30-year moving average technique to estimate the market risk premium based on CAPM (Price ke MA30) as well as the estimated fundamental price based on the DDM and the cost of equity estimated using the expanding window technique to estimate the market risk premium based on CAPM (Price ke ExpWin).

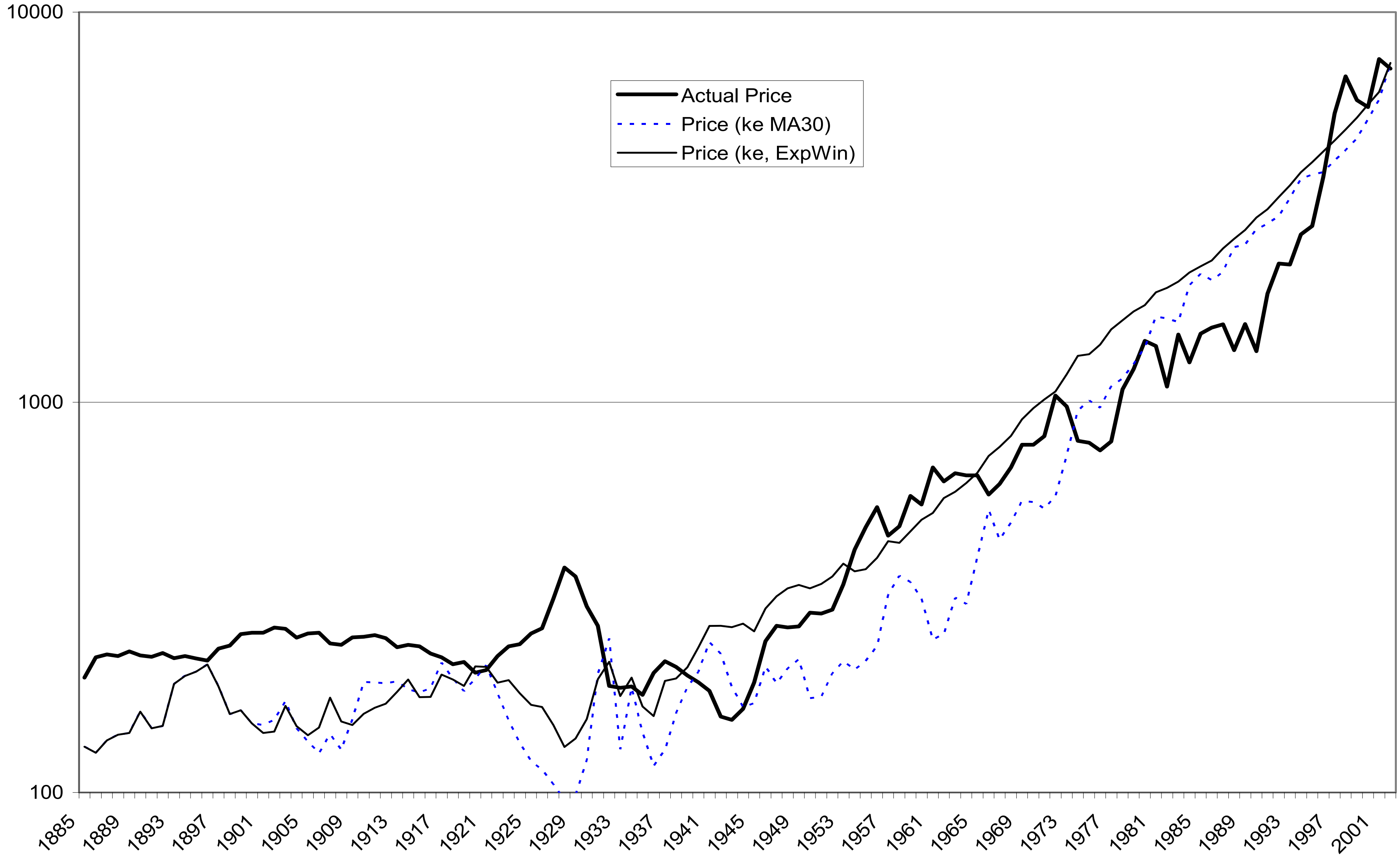

Figure 4

Actual and Estimated Fundamental Bank of Montreal Prices Based on the Gordon Growth Model (GGM), 1885-2003

Comparison of the actual and fundamental or estimated share price for Bank of Montreal. We present the actual Bank of Montreal price (Actual Price) and the estimated fundamental price based on the GGM in equation (4) with the cost of equity estimated using the 30-year moving average technique to estimate the market risk premium based on CAPM and GNP growth rates (Price ke MA30) and the estimated fundamental price based on the DDM and the cost of equity estimated using the expanding window technique to estimate the market risk premium based on CAPM and GNP growth rates (Price ke ExpWin).

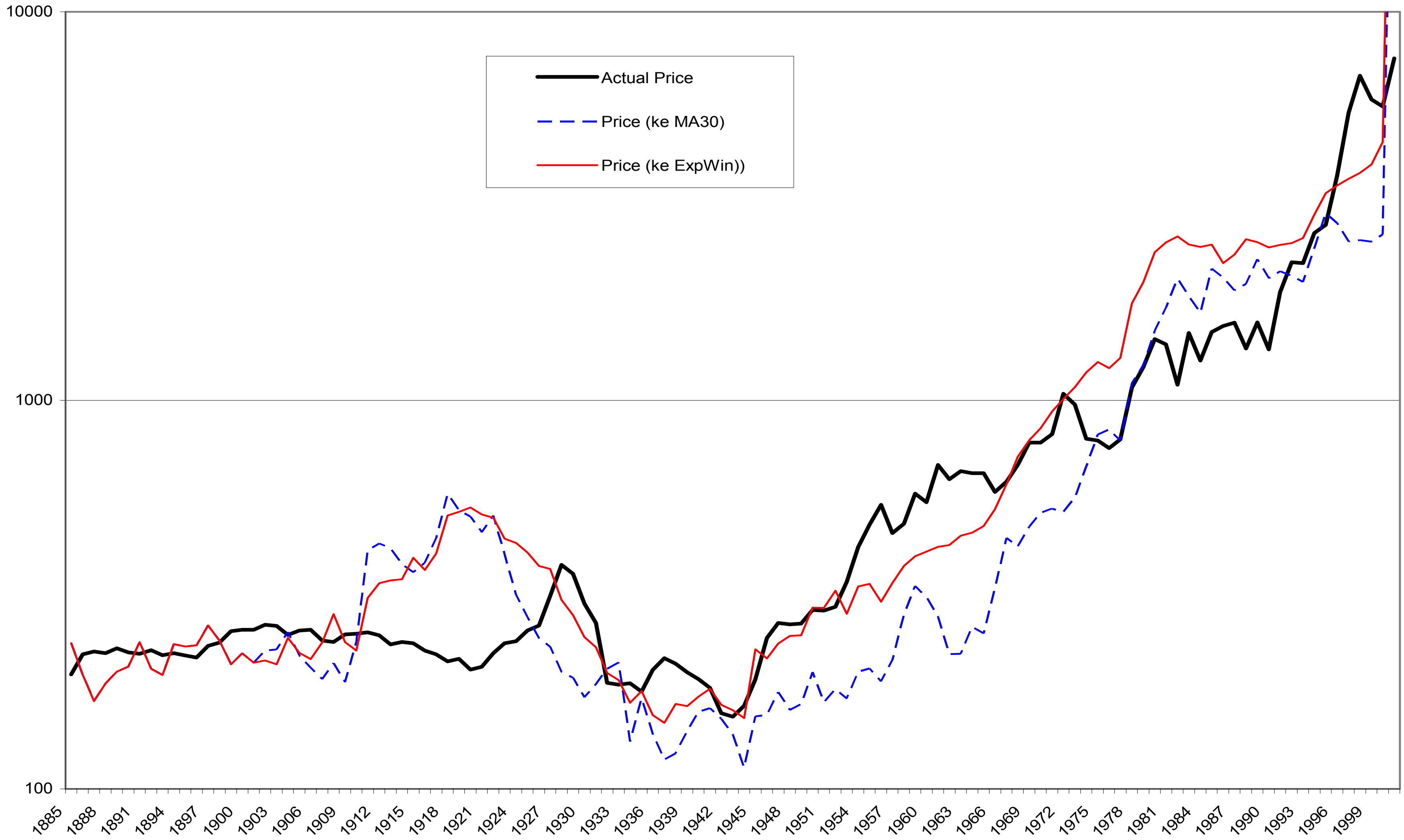

Figure 5:
Fed Model: BMO Earnings Yield and the Long-term Government Bond Yield.

Graph of the earnings yield for the Bank of Montreal calculated as the Earnings per Share divided by the share price compared to the return on the 10 year Treasury Bond in each year for the period from 1885 to 2003.