Optimal Outsourcing Strategies when Capacity is Limited

ABSTRACT
Outsourcing the production of selected components to competitors is becoming more common among original brand manufacturers (OBM). Increasing attention to outsourcing by OBMs and growing demand in many markets result in capacity allocation conflicts for the contract manufacturers. In this paper, we consider a scenario in which the OBM decides whether to outsource to a third-party supplier or to a Competitive Contract Manufacturer (CCM) who has the option of producing a competing product and is capacitated. The customers are heterogeneous and the competing products are horizontally differentiated. The CCM first chooses the wholesale price and whether or not to sell a competing product to the customers. Next, the OBM decides how much to outsource to the CCM, and finally the retail prices are decided. We find that when capacity increases, demand may decrease while retail price may increase. Moreover, the CCM can be worse off from having more capacity, even when capacity is available for free. Our results also show that demand may increase when competition in the final product market becomes more intense. Finally, we find that the value of having a third-party supplier to produce the component decreases in the intensity of competition in the final product market.

Subject Areas: Competitive Contract Manufacturer, Price Competition, and Capacity Allocation Conflict.

INTRODUCTION
TPV technology (TPV), the largest electronic manufacturer of computer monitors, both sells monitors under its own brand (AOC and Envision) in final product market and is a supplier to Philips which sells monitors under the Philips brand competing with TPV’s AOC and Envision. The demand for monitors is beyond the capacity of TPV, and thus TPV has decided to reduce production of its own brand in order to satisfy outsourcing orders of Philips (Wang, 2008).

Outsourcing the production of some of the components to competitive contract manufacturers (CCM, e.g., TPV) is becoming more common among original brand manufacturers (OBM). However, growing demand results in capacity allocation conflicts for these CCMs. For example, Apple sources its NAND Flash memory requirement from Samsung (Kim, 2012). However, as smartphones become more popular, Samsung is having a difficult time
fulfilling the demand. Such capacity allocation conflict “would be bad for Apple if Samsung were forced to choose between Apple and itself in case of a supply shortage at its factories” (Forbes, 2013). As yet another example, Franz Inc. is a contract manufacturer in producing home décor accessories (e.g., tableware, vases and jewellery) for OBM s such as Enesco and Lenox. In 2002, it started to sell products under its own brand while continuing to supply for the OBM s. It reached its capacity limit due to increasing orders from OBM s, and eventually in 2005, Franz decided to prioritize the production of its own brand products ahead of others (Yan, 2013). When the CCM has a limited capacity, the OBM can influence the CCM’s output to the final product market by using a portion of the CCM’s capacity, thereby mitigating competition in the final product market. On the other hand, precisely because of this reason, the CCM would set a higher wholesale price. Thus, the introduction of a capacity constraint adds some interesting trade-offs to the firms. As both cooperation among competitors and capacity shortages become more often, study of the interaction between these phenomena and the resulting impacts becomes more relevant and interesting.

In this paper, we analyze an OBM's outsourcing strategies when the CCM is capacitated. In particular, the OBM does not produce a critical component of its product in-house (e.g., monitor in TPV’s example, NAND Flash memory in Samsung’s example and home accessory in Franz’s example) and thus has to decide whether to outsource the production of that component to the CCM or to a third-party supplier. Furthermore, the CCM is capacitated in the production of the critical component, and it must decide whether or not to sell products to customers under its own brand. Moreover, the CCM must decide the wholesale price of the component to compete with other third-party suppliers. In order to represent the motivation examples of the study (e.g., Philips vs. AOC) we assumed that the customers are heterogeneous and the
competing products are horizontally differentiated. Despite the fact that previous research has studied the supply chain partnership between competitors (e.g., Venkatesh, Chintagunta, & Mahajan, 2006; Xu, Gurnani, & Desiraju, 2010; Wang, Niu, & Guo, 2013) and capacity allocation problem (e.g., Gupta & Wang, 2007; Mallik, 2007; Ülkü, Toktay, & Yücesan, 2007), there is no research that studies the capacity allocation problem when competitors are supply chain partners (coopetitors). The goal of this paper is to merge these two streams of literature by considering how the following factors affect the outsourcing strategy and the firms’ profitability:

1. The CCM’s capacity
2. Competition in the final product market (between the OBM’s product and the CCM’s product)
3. Competition in the component market (between the CCM and the third-party supplier)

These are our interesting findings:

- The CCM has the option of not selling final products to customers, but rather using the capacity to be the OBM’s sole supplier, so that the CCM and the OBM are monopolies in the component market and the final product market, respectively. However, we find that, the firms would always forgo this opportunity. In particular, when capacity is very small, the firms would choose to be competitors.

- When capacity increases, one might expect that demand would (weakly) increase while retail price would (weakly) decrease. We find that this intuition may not be true.
Even when capacity is available for free, the CCM can be worse off from having more capacity. This impact of profit-decreases-in-capacity is always larger when both firms are coopetitors than when both firms are competitors.

The demand for the CCM’s product may increase when competition in the final product market is more intense (e.g., when the competing products become more substitutable).

Even though the CCM has less incentive to allocate its capacity to produce components for the OBM when the intensity of competition in the final product market increases, the value of having a third-party supplier to produce the component is minimal.

This paper is organized as follows. In the next section, we review the related literature. We then present the mathematical model and the analytical results in the next two sections. In the following section, we analyze the value of competition in the component market by considering a scenario where the CCM produces a proprietary component for OBM that no other third-party is capable to produce. Finally, we conclude the paper. The details of the derivation of the equilibriums and the proofs of the results are presented in the appendices.

LITERATURE REVIEW
The first related stream of literature is on the topic of capacitated contract manufacturer. Gupta and Wang (2007) study capacity allocation problem of a contract manufacturer that can accept two types of orders: high volume contractual orders as well as one time transactional orders. They use Markov decision process in order to evaluate the optimal decisions of a contract manufacturer in each period. They show that the optimal acceptance policy is a threshold policy that specifies an accept-up-to level. They also find that it might be optimal for a contract manufacturer to serve only transactional orders when capacity is tight. Ozkan and Wu (2009)
study the capacity allocation problem of a contract manufacturer in the high-tech industry where different orders might or might not be able to share the same capacity. They design a make-to-stock and make-to-order mechanisms to find the optimal production and capacity allocation levels for each period over planning horizon. They show the conditions to find the fixed capacity allocation level between two different orders. Ülkü et al. (2007) also study a contract manufacturer with limited resources who gets orders from different original equipment manufacturers. They investigate who should take the risk of under or over investment on the productive resources due to demand uncertainty. They find premium-based schemes that maximize the supply chain profit despite information asymmetry on demand forecasts. Mallik (2007), however, studies the impact of uncertain demand and high production lead time on the capacity allocation problem of a semiconductor. He uses a game theoretical model and designs a bonus scheme and an allocation rule mechanism that ensures truthful demand forecast statistics from all product managers. His work is related to Cachon and Lariviere (1999) who study the capacity allocation rules of a single supplier with limited capacity that sells to several retailers who are privately informed of their optimal stocking levels. Cachon and Lariviere (1999) find that truth telling might not be a universally desirable goal under which the manufacturer will choose a lower capacity level. However, this stream of literature assumes that the contract manufacturer uses its capacity to supply to a downstream retailer or final brand manufacturer and therefore does not consider the potential use of the capacity by contractor to produce its own competing brand.

The second stream of related literature is about competing firms that have only limited amount of capacity to produce their products. Gelman and Salop (1983) use a game theoretical model to study the entry of a firm that could choose its capacity before entering to a competition
with the incumbent. They show that a monopoly with unlimited capacity will not be able to deter the entrance of a new firm with the capability to choose its initial capacity level. Therefore, the entrant chooses to limit its capacity so that the incumbent does not find it profitable to undercut its price. They call this a “judo economics” scenario where the incumbent’s unlimited capacity is a disadvantage for it. In addition, they argue that even if capacity is free, the entrant is better off to limit its capacity. We also find similar results where the OBM does not want to be a monopoly in the market when the CCM’s capacity is small. This is due to the fact that the existence of a low-output CCM will result in having higher market prices than the monopoly prices, and this encourages the OBM to forgo the opportunity of being a monopoly in the final product market, as in Gelman and Salop (1983). Osborne and Pitchik (1985) characterize the Nash equilibria in a duopoly with capacitated Bertrand competition. They find that having a limited capacity could be beneficial for the small firm as the reduction in competition due to smaller capacity may offset the lower output levels of the small firm. We also show that the CCM’s profit may decrease in capacity because of higher market competition. Biglaiser and Vettas (2004) study a dynamic pricing game of capacitated firms. They find no pure-strategy subgame perfect equilibrium and they show that the market share would likely be maximally asymmetric. Martínez-de-Albéniz and Talluri (2011) use a game theoretic model to study dynamic price competition in an oligopoly where demand for each period is uncertain. They characterize the equilibrium conditions as “competitive bid-price” where each firm has a reservation value depending on its capacity level, and they show that the firm with lower value would offer a market price equal to other firm’s reservation value. The difference of our research with this stream of literature is that we assume that one of the firms does not produce its products in-house and has the option of outsourcing to its capacitated competitor. Therefore, we extend
the results of this stream of literature to the setting where the competitors might share the limited capacity.

The third stream of literature examines the scenario of partnership of the rivals. Venkatesh et al. (2006) study the optimal strategies of a manufacturer of propriety component brands (MPCB). The MPCB can choose to use the components exclusively for its own brand products, to supply the components to an original brand manufacturer, or to use the components in its own brand as well as supplying them to its competitor. They use a game theoretical model and find that even if final products are highly substitutable, it is optimal for the MPCB to use components for its own brand products as well as supplying them to its competitor. Xu et al. (2010) extend the findings of Venkatesh et al. (2006) by studying the effect of production cost differences and uncertain product differentiation on the optimal supply chain structure. They find that when the proprietary component manufacturer invests in component branding, the preference between supplying to competitor and being only a component supplier depends on the impact of the investment. Both papers show that the proprietary component manufacturer chooses to be a monopoly in the final product market only when the two products are almost perfect substitutes.

Lim and Tan (2010) study a two-period model to find the optimal outsourcing strategies of an original equipment manufacturer that might choose to produce its products in-house or outsource to a contract manufacturer. They evaluate the original equipment manufacturer’s make, buy, and make-and-buy decision under different learning rates of the contract manufacturer. They find that high customer brand preference for the original equipment manufacturer’s product might deter the contract manufacturer from introducing its own brand. Wang et al. (2013), on the other hand, study the advantage of being the first mover in a Cournot
competition where the original equipment manufacturer has the option of outsourcing to its competitors as well as outsourcing to other third-parties with no competing products in the market. Therefore, they extend the results of the previous studies to a setting where CCM competes with other third-party contractors to get outsourcing orders from original equipment manufacturer. They show that, when the price of the component of the third-parties is high, CCM will price its components sufficiently low to keep the original equipment manufacturer in the market. Wang et al. (2013) assumes that wholesale price of the CCM has to be at most equal to other third-party contractors’ wholesale price, or else the original equipment manufacturer would choose to outsource to a cheaper third-party. However, we show that in a capacitated setting OBM might want to pay higher wholesale prices for CCM’s components because it can reduce competition in the final product market. Pun (2014), using a game theoretical model, studies the outsourcing strategies of an OBM that also can exert effort to improve its production process for higher customer valuation. He finds that it might be optimal for the OBM to outsource to its competitor and let the competitor exert effort even if it has higher cost. This stream of literature assumes that the contract manufacturer has unlimited capacity to supply its own products as well as outsourced orders. We reveal some counterintuitive findings when considering the capacitated contract manufacturer structure.

To the best of our knowledge, we are the first to examine how capacity affects the supply chain structure in an oligopolistic competition when one firm might outsource to its competitor.

**MODEL DEVELOPMENT**

We consider a scenario where an OBM (firm O) must outsource the production of a critical component. For example, the component can be monitor for the TPV example, NAND Flash memory for the Samsung example and home accessory for the Franz example. We assume that
the production cost of this component is normalized to zero, so that firm O’s outsourcing decision is driven by the structural differences between outsourcing to competitor and outsourcing to third-party supplier and not by the difference in production cost. Moreover, for simplicity, we assume that the final product consists of this component only. There are two potential suppliers: The first component supplier is a CCM (firm C). It sells components to firm O at a wholesale price $w$; it also decides whether or not to sell products under its own brand to customers. Moreover, firm C’s production capacity of the component is $k$. In order to examine the effect of capacity, similar to Gupta and Wang (2007), we assume that firm C produces everything in-house and does not outsource to other third-parties when facing capacity shortage. The second component supplier is a third-party supplier (firm T) that does not have the option of producing a competitive product under its own brand. Similar to other related literature on contract manufacturing (e.g., Jeannet, 2009; Wang et al., 2013), we assume that there are many identical and independent third-party suppliers competing to be firm O’s supplier, and firm T is one of these suppliers. This assumption is in line with many industry practices. As an example, despite the fact that Samsung has 30% market share in NAND Flash memory market, there are many other non-competitive suppliers (such as Toshiba, SanDisk and Intel) that apple can outsource this critical component (DRAMeXchange, 2014). Due to the intense competition among these suppliers, firm T’s wholesale price is exogenously determined as the equilibrium market price in a competitive market, which is normalized to zero. Moreover, since firm O can outsource to another of these suppliers whenever capacity of one of them is met, we do not consider a capacity limit for firm T.

Firm O decides the proportion of its component demand to be allocated to firm C ($\gamma \in [0,1]$), and the remaining component demand ($1 - \gamma$) will be allocated to firm T. An
alternate interpretation of this decision is that firm O decides how much (in absolute term) to outsource to firm C. There are three outsourcing strategies for firm O: (1) Firm O does not outsource to firm C (i.e., $\gamma = 0$), so firms O and C are pure competitor; (2) Firm O single-sources to firm C (i.e., $\gamma = 1$), and (3) firm O multi-sources to firms C and T (i.e., $0 < \gamma < 1$).

The two firms are supply-chain partners and competitors when $\gamma > 0$ if firm C also sells products under its own brand. Similar to Gupta and Wang (2007), we assume that firm C is obligated to satisfy the orders for firm O, so firm C needs to set the wholesale price $w$ strategically in order to better utilize its capacity. To illustrate, if firm C wants to reserve more capacity to produce for products under its own brand, it can set a high wholesale price to deter firm O from ordering too much.

None of the firms that motivate this study is able to produce products that are uniformly better than the other, so we consider a horizontally differentiated model that is similar to the one presented in Venkatesh et al. (2006) and Xu et al. (2010). In particular, the customers have reservation price $R$ for a product. The products of firms O and C are located at an exogenously specified distance $M$ apart. The two products are more substitutable and hence competition is intense when $M$ is small. The length of the Hotelling line is sufficiently larger than $M$ such that all customers located between the two firms would buy but not all customers located outside the two firms would buy. Each customer incurs a disutility of $t$ per unit distance and will only buy the product that gives him/her the higher positive utility. Despite the fact that all the results can be driven for more general form, for expositional convenience, we assume $t = 1$ and $R = 1$. Customers would have zero utility when not making a purchase. When buying a product from firm $i \in \{O, C\}$ at retail price $p_i$, a customer that is $d$ away from firm $i$ would have utility:

$$U = 1 - d - p_i$$  \hspace{1cm} (1)
The demand of firm \( i \) (\( D_i \)) can be derived from the customer’s utility function (refer to Venkatesh et al., 2006; Xu et al., 2010 for the solution approach). Then the profit function of firm O is as follows:

\[
\pi_o = (p_o - w) \gamma D_o + p_o (1 - \gamma) D_o
\]  

The two parts of \( \pi_o \) are the profits from selling products containing firm C’s and from selling products containing firm T’s components, respectively. When firm C sells products under its own brand, the profit and the capacity constraint are

\[
\pi_c = w \gamma D_o + p_c D_c
\]

\[
\gamma D_o + D_c \leq k
\]

When firm C does not sell products under its own brand, the profit and the capacity constraint of firm C are

\[
\pi_c = w \gamma D_o
\]

\[
\gamma D_o \leq k
\]

As commonly used in the related literature (e.g., Cui, Raju, & Zhang, 2008; Wang et al., 2013) and consistent with many industry practices (e.g., Foxconn, Asustek), we assume that firm C first sets the wholesale price \( w \) and then firm O decides which supplier to outsource to given the wholesale prices. Therefore, we consider two levels of competition: competition in the component market between firms C and T, and competition in the final product market between firms O and C. The game sequence is as follows.

1) Firm C decides \( w \) and whether or not to have its own product.
2) Firm O decides $\gamma$.

3) Firm O decides $p_0$. If applicable, firm C decides $p_C$.

We use backward induction to find the equilibrium solutions.

**EQUILIBRIUM CONDITIONS**

In this section we first present the equilibrium strategy in Proposition 1, and then we perform sensitivity analyses with respect to firm C’s capacity in Proposition 2 and with respect to the substitutability of the products in Proposition 3. In order to derive the equilibrium solution, we use the Karush-Kuhn-Tucker conditions to consider firm C’s capacity constraint. We separate the optimization problem into two cases: 1) *Binding capacity equilibrium* where firm C uses all of its capacity and 2) *Non-binding capacity equilibrium* where firm C has some unused capacity. Details of the derivation of the equilibrium are presented in the Appendix. For expositional convenience, define $k_{12} \equiv \frac{3(2+M)}{10}$, $k_{23} \equiv \frac{7(2+M)}{15}$ and $k_{34} \equiv \frac{3(2+M)}{5}$. Note that these thresholds describe the capacity, and are named such that the subscripts describe where the thresholds are located in Figure 2. For example, $k_{23}$ is located at the boundary of regions II and III. Lemma 1 presents the wholesale price threshold used for the equilibrium solution.

**Lemma 1:** Define $\bar{w}$ such that firm O is better off outsourcing to firm C (i.e., $\gamma > 0$) if and only if $w \leq \bar{w}$, where

\[
\bar{w} = \begin{cases} 
\frac{1}{68} (12 - 3k + 6M) & k \leq k_{12} \\
\frac{1}{20} (6 - 5k + 3M) & k_{12} < k < k_{34} \\
0 & k \geq k_{34}
\end{cases}
\]  

(7)
The value $\bar{w}$ is illustrated in Figure 1. When firm O single-sources from firm C and when firm C sells its own product, the interior solution (in the absence of capacity constraint) for the demand is $k_{12} \equiv \frac{3(2+M)}{10}$ for each firm. Therefore, when firm C has sufficient capacity to supply for both firms ($k \geq k_{34} \equiv \frac{3(2+M)}{5}$), the capacity constraint is not binding. Hence, Firm O will only accept a wholesale price from firm C that is not higher than that of firm T, which is zero. On the other hand, when firm C does not have sufficient capacity to produce for both firms ($k < k_{34}$), firm O can reduce the supply of firm C’s final product (and hence the competition at the final product market is mitigated) by outsourcing to firm C. Because of this benefit, firm O is willing to accept a wholesale price that is higher than that provided by firm T (i.e., $\bar{w} > 0$). $\bar{w}$ decreases in the capacity $k$, because the impact of reducing competition at the final product market is larger when $k$ is small.

Even though outsourcing to the competitor would have an advantage of reducing the competitor’s output, the wholesale price $\bar{w}$ that firm O is willing to accept is small when competition in the final product market is intense ($M$ is small). This is because the retail prices of both products are small when the two products are very substitutable ($M$ is small), so to maintain a positive margin, firm O would only accept a small wholesale price.
Proposition 1 presents the optimal strategies of the two firms. Denote the optimal solution with superscript “*”. The optimal prices, demands and profits are available in table A.

**Proposition 1:** The optimal strategy is such that firm C sells products to customers and

1) If \( k < k_{23} \), firm O single-sources from firm T (\( \gamma^* = 0 \)) and \( w^* > \bar{w} \).

2) If \( k \geq k_{23} \), firm O single-sources from firm C (\( \gamma^* = 1 \)) and \( w^* = \bar{w} \).

The capacity constraint is binding if and only if \( k \leq k_{12} \) or \( k_{23} \leq k \leq k_{34} \).

Figure 2 illustrates the equilibrium solution. When firm C has plenty of capacity (region IV), it has sufficient capacity to produce for both firms. Firm O will only accept a wholesale price from firm C that is not higher than that of firm T. Therefore, firm C sets \( \bar{w} = 0 \) (cf. Lemma 1) and firm O outsources to firm C. This is the region where the literature that studies supply-chain partnership with competitor is focusing on (e.g., Venkatesh et al., 2006; Xu et al., 2010; Wang et al., 2013).

**Figure 2:** Optimal outsourcing strategies.
Firm C’s capacity is intermediate at region III. When firm O outsources to firm C, it can reduce the supply of firm C’s product and the competition in the final product market can be mitigated. Therefore, firm O would outsource to firm C and firm C sets a non-negative wholesale price \( w = \bar{w} \). The outsourcing strategy in this region could explain how Philips caused TPV to reduce its own brand output by outsourcing its monitor production to TPV.

Since \( \bar{w} \) weakly decreases in the capacity of firm C (cf. Lemma 1), one might expect that when firm C has a low capacity (regions I and II where firm C can charge a high wholesale price to firm O), instead of using the capacity to produce for its own product, firm C is better off using all capacity to supply to firm O so that firms O and C can be monopolies at the customer and at the component markets, respectively. Interestingly, we find that firms O and C would rather be pure competitors in these two regions. This is because on one hand firm C can sell its product to the customers at a high retail price and hence it requires a high wholesale price if it were to use the capacity to produce for firm O’s product instead of to produce for its own product. On the other hand, competition between suppliers C and T provides a limit about how high the wholesale price that firm O is willing to accept. We find that the wholesale price that justifies firm C to use the capacity to produce components for firm O’s product is higher than the wholesale price that firm O is willing to accept (i.e., \( w^* > \bar{w} \)). Therefore, both firms would forgo the opportunity of becoming monopolies at the component and at the final product market and would rather be pure competitors. Moreover, when firm C sells final product to customers when it has small capacity, it will price its products high, which in turn will allow firm O to price its product higher than its price in the monopoly market. Gelman and Salop (1983) have similar results where they find that when the new entrant is capacitated, it is not profitable for the
incumbent to be a monopoly in the market; we extend their results to the case where competitors are supply chain partners.

At region II, firm O does not outsource to firm C even though firm C has some unused capacity. This is because firm C being firm O’s supplier would reduce the capacity to produce for its own products, but the gain from component sales to firm O would not compensate for the loss from the reduction in final product sales. Therefore, firm C would set a high wholesale price (i.e., \( w^* > \bar{w} \)) to discourage firm O from outsourcing to firm C. This result can also explain the market choices of some competitive CMs like Franz who prioritize capacity to their own brands and don’t accept outsourcing contracts when facing capacity allocation conflicts (Yan, 2013).

Firm O would never multi-source to both suppliers (i.e., \( 0 < \gamma < 1 \) is never true) because it wants to outsource as many components as possible to firm C if the wholesale price is less than \( \bar{w} \). Therefore, the only possible scenario that might lead to multi-sourcing is when firm C’s capacity is not enough to satisfy all firm O’s demand and firm C sets a wholesale price less than \( \bar{w} \). However, we show that firm C prefers final product sales over component sales when its capacity is low, so firm C would set a high wholesale price. Therefore, firm O would never multi-source.

Proposition 2 presents the impact of capacity to demand, price and profit.

**Proposition 2:** Define \( k_1 \equiv \frac{7(2+M)}{24}, \quad k_3 \equiv \frac{8(2+M)}{15} \) (as thresholds located inside region I and III respectively), \( k_{23}^{-} \equiv k_{23} - \varepsilon \) and \( k_{23}^{+} \equiv k_{23} + \varepsilon \), where \( \varepsilon \) is a small positive number.

**a.** Firm C’s demand and the total demand may decrease in capacity: \( D_C^*(k_{23}^{-}) > D_C^*(k_{23}^{+}) \) and \( D_O^*(k_{23}^{-}) + D_C^*(k_{23}^{-}) > D_O^*(k_{23}^{+}) + D_C^*(k_{23}^{+}) \).
b. Firm i’s price may increase in capacity: \( p_i^*(k_{23}^-) < p_i^*(k_{23}^+) \).

c. Firm C’s profit may decrease in capacity:

\[
\begin{align*}
\frac{\partial \pi_C^*}{\partial k} &< 0 \iff k_1 < k \leq k_{12} \text{ or } k_3 < k \leq k_{34} \\
\left| \frac{\partial \pi_C^*}{\partial k} \bigg|_{k=k_1+\Delta} \right| &< \left| \frac{\partial \pi_C^*}{\partial k} \bigg|_{k=k_3+\Delta} \right| \text{ for all } \Delta > 0
\end{align*}
\]

When firm C’s capacity \( k \) increases, one might expect that prices would decrease while demands would increase. We find that this intuition does not hold when the capacity of firm C is around \( k_{23} \) (cf. Propositions 2a and 2b). This is because as \( k \) increases, the strategy changes from the two firms being pure competitors (region II) to being coopetitors (region III). Therefore, firm C would shift some of its capacity to produce components for firm O and so its demand \( D_C^* \) decreases and price \( p_C^* \) increases. Venkatesh et al. (2006) find that firms would set higher prices under coopetition relationship. We extend their finding to a capacitated system by showing that firm C sets higher price because, in addition to the two firms being coopetitors, firm C uses some of the capacity to produce for firm O and so it produces fewer units for itself. Firm O also sets a higher retail price \( p_O^* \) because it shifts from using a cheaper supplier (firm T) to a more expensive supplier (firm C) so it sets a higher retail price to maintain the margin.

When firm O outsources to firm C and firm C has sufficient capacity to produce for both firms (region IV), or when firm O outsources to firm T and firm C has sufficient capacity to produce for itself (region II), firms’ profits are not affected by the capacity level. However, when capacity is binding, firm C may be worse off from having more capacity, even when capacity can be available for free. This is because in regions I and III, the retail prices of both firms would decrease when firm C has more capacity, so competition is more intense. We find that the impact
of a decrease in retail prices is larger than the impact of an increase in demand, so the profit of firm C decreases. In other words, selling the extra output requires firm C to lower its market price which results in lower overall profit. This can be interpreted as the cost of selling the extra output to the firm. Furthermore, the second part of Proposition 2c shows that when firms O and C are cooperating as supply-chain partners, the impact of profit-decreases-in-capacity is larger compared to the case where firms are only competitors. This is because firms would set higher prices under the coopetition scenario than under the competition scenario. When capacity increases, the decrease in price under the coopetition scenario is larger than that under the competition scenario, so the decrease in profit is larger. This finding illustrates the importance of considering the firm’s capacity constraint when competitors are cooperating as supply-chain partners.

Proposition 3 presents the impact of product substitutability to the demand of firm C.

**Proposition 3:** Firm C’s demand may increase in the intensity of competition in the final product market: \( \frac{\partial D_C^*}{\partial M} < 0 \iff k_{23} \leq k \leq k_{34} \).

Proposition 3 shows that the demand for a product can increase even when the two products become more substitutable \((M \text{ decreases})\). This is because firm O’s demand decreases when \( M \text{ decreases} \), so firm C allocates lesser capacity to produce for firm O’s product. Firm C would have more capacity to produce for product under its own brand, so it would set a lower price \( p_C^* \) to its product, leading to a higher demand \( D_C^* \).
VALUE OF COMPETITION IN COMPONENT MARKET
Contract manufacturers sometimes have the proprietary rights to produce the component, but after the patent has expired other suppliers can also produce it. For instance, Qualcomm was the proprietary supplier of the CDMA chips for cell phone producer, and the expiration of its CDMA patents ended Qualcomm’s control over CDMA (Mock, 2005, p. 184), resulting in an increase of competition in the cell phone chip manufacturing market. The purpose of this section is to evaluate the impact of competition in the component market. In particular, we assume in the main model that the component is not proprietary such that firm O has multiple potential suppliers (firms C and T). In this section, we consider a benchmark in which the component is of proprietary nature, and firm C is the only supplier that can produce the component. The problem becomes similar to that presented in Venkatesh et al. (2006) and Xu et al (2010), and we expand their studies by considering a capacitated system. Firm C deploys one of the following strategies: (1) monopoly - not supplying component to firm O (e.g., sets a very high wholesale price) such that firm C is the monopoly in selling the final product, (2) component supplier - be a supplier of firm O but not entering into the final product market, and (3) coopetitor – supplies component to firm O and sells final products to customers.

When firm C does not supply components to firm O (monopoly), firm O would have zero profit, and firm C’s optimization problem is

\[ \pi_c = p_c D_c \]  \hspace{1cm} (8)

\[ \text{s.t. } D_c \leq k \]  \hspace{1cm} (9)

When firm C supplies component to firm O and does not sell final product in final product market (component supplier), firm C’s optimization problem is:
\[ \pi_C = w \ D_O \]  \hspace{1cm} (10)

\[ \text{s.t.} \ D_O \leq k \]  \hspace{1cm} (11)

When firm C sells components to firm O and also sell final product in final product market (coopetitor scenario), its profit is:

\[ \pi_C = w \ D_O + p_C \ D_C \]  \hspace{1cm} (12)

\[ \text{s.t.} \ D_O + D_C \leq k \]  \hspace{1cm} (13)

Under the component supplier and the coopetitor scenarios, the profit of firm O is:

\[ \pi_O = (p_O - w) \ D_O \]  \hspace{1cm} (14)

The game sequence under the benchmark is as follows:

1) Firm C decides on its strategy (monopoly, component supplier, coopetitor).

2) If applicable, firm C decides \( w \).

3) If applicable, firm C decides \( p_C \) and firm O decides \( p_O \).

We use backward induction to find the equilibrium solutions. The derivation of equilibrium is presented in the appendix.

Denote the optimal profit of firm O under the benchmark to be \( \pi^{B}_O \), and define the value of competition to firm O be \( V_O = \pi^{*}_O - \pi^{B}_O \). Then Proposition 4 examines the impacts of capacity and competition in the final product market to the value of competition in the component market. (The value of competition from the perspective of firm C is simply the reverse of that from the perspective of firm O.)
**Proposition 4:**

a. The value of competition in the component market decreases in capacity: $\frac{\partial v_o}{\partial k} \leq 0$.

b. The value of competition in the component market decreases in the intensity of competition in the final product market: $\frac{\partial v_o}{\partial M} \geq 0$.

When the CCM’s capacity decreases, the wholesale price would increase significantly under the benchmark because the CCM is the monopoly in the component market. On the other hand, the wholesale price would be relatively insensitive to the capacity under the main model because of competition in the component market. Therefore, the value of competition in the component market is large when capacity decreases.

When the competing products are highly substitutable (i.e., small $M$), firm C has less incentive to allocate its capacity to firm O, so one might expect that the value of competition in the component market is large. However, we find that the opposite impact holds. Consider the case where firm C supplies component to firm O under the benchmark. (Otherwise, firm O has zero profit, so the comparison is trivial.) Firm O’s profit is relatively insensitive to the product substitutability under the benchmark because firm C would set a wholesale price to extract as much profit from firm O as possible when it is the proprietary component supplier. On the other hand, under the main model, the wholesale price would be relatively insensitive to the product substitutability because of competition in the component market. As firm O can gain more when the product becomes less substitutable under the main model, the value of competition in the component market increases in $M$. 

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CONCLUSION
In this paper, we study the impact of capacity on the optimal channel structure in a contract manufacturing business. The customers are heterogeneous with different preferences regarding the products that are horizontally differentiated. We show that capacity limitation, which is a commonly experienced conflict among contract manufacturers, can have nontrivial impacts. In particular, when firms are supply chain partners the CCM might reduce its own product output in order to fulfill OBM’s outsourcing orders. We also show that firms’ prices might increase and demand might decrease as capacity increases. Interestingly, we find that CCM’s profit may decrease in its capacity and this deterioration is more severe when firms are supply chain partners. Lastly, we show that the value of competition in the component market to the OBM is small when the two products are highly substitutable.

We use a stylized model to study the dynamics of the firms’ optimal decisions, and there are several limitations to our model. We assume that the demand is deterministic and all firms have full information regarding their demand models. However, in a more realistic situation firms would only have a forecast of their demand. Therefore, a possible avenue of future research is to examine the impact of demand uncertainty on the capacity allocation problem of a CCM. In this scenario, CCM would face a challenge of having capacity shortage or overage when accepting orders from the OBM. Moreover, since the CCM will produce a competitive product, the OBM might want to order more than its true demand forecast to reduce competition in the final product market. Finally, in our model we only consider one competitive contract manufacturer that can produce a competitive product. It would be interesting to consider multiple strategic CM’s with the option of producing their own brand products.
REFERENCES


APPENDIX

Derivation of the equilibrium

In this section we derive the equilibrium conditions of the 3-stage game defined in “Model Development” section. From equation (1): \( D_o = \frac{1}{2} (2 + M - 3p_o + p_c) \); \( D_c = \frac{1}{2} (2 + M - 3p_c + p_o) \). We only consider the region where \( 0 < M < 2 - p_o - p_c \) such that the two products are competing. Otherwise, these are two separate markets and we are not able to examine the cooperation between competitors.

1 - Stage 3 of the game

Both firms’ profit function is concave in its price (i.e., \( \frac{d^2 \pi_i}{dp_i^2} < 0 \)), so FOC gives: \( p_o^*(p_c) = \frac{1}{6} (2 + M + p_c + 3w\gamma) \). We use KKT conditions to consider the capacity constraint that result in two optimal pricing strategy for firm C: 1) Non-Binding Capacity Constraint: \( p_c^*(p_o) = \frac{1}{6} (2 + M + p_o + w\gamma) \). 2) Binding capacity constraint: \( p_c^*(p_o) = \frac{2-2k+M+p_o+(2+M-3p_o)\gamma}{3-\gamma} \). Then, we can solve for the optimal prices under each capacity condition: Case 1 - Non-Binding Capacity Constraint: \( p_o^* = \frac{1}{35} (14 + 7M + 19w\gamma), p_c^* = \frac{1}{35} (14 + 7M + 9w\gamma) \); For case 1, we need \( \gamma D_o + D_c < k \), so we have condition (1): \( \frac{1}{70} (42(1 + \gamma) + 21M(1 + \gamma) - 8w\gamma(1 + 6\gamma)) < k \). Case 2 - Binding Capacity Constraint: \( p_o^* = \frac{8-2k+4M+3w(3-\gamma)\gamma}{17-3\gamma}, p_c^* = \frac{7(2+M)+3(2+M+w)\gamma-12k-9w\gamma^2}{17-3\gamma} \).
2 - Stage 2 of the game

Case 1 - Firm O’s profit function is strictly convex (i.e., $\frac{d^2 \pi_0}{d \gamma^2} > 0$) which implies that optimal allocation ratio $\gamma^*$ is at some extreme point (either zero or the largest feasible allocation ration between zero and one depending on the available capacity of firm C). For firm O to set a non-zero ratio (i.e., $0 < \gamma^* \leq 1$), the resulting profit should be larger than its profit when $\gamma = 0$ (i.e., $\frac{3}{50}(2 + M)^2 \leq \frac{3(14 + 7M - 16wy)^2}{2450} \iff \frac{14 + 7M}{19y} \leq w \leq 0$), such that firm O will outsource as much component as it can to firm C depending on firm C’s available capacity.

Case 2 - Firm O’s profit function is neither convex nor concave. Nevertheless, there is only one root to the FOC condition ($\frac{d \pi_0}{d \gamma} = 0 \rightarrow \gamma = \frac{4-k+2M}{4w}$) which is a minimizer ($\frac{d^2 \pi_0}{d \gamma^2} \mid \gamma = \frac{4-k+2M}{4w} > 0$). However, this critical point might or might not be in the feasible region ($0 \leq \gamma \leq 1$) depending on the values of other parameters. Though, since there is only one stationary point, we can claim that the profit function is maximized at extreme points if the stationary point is in the feasible region or the profit function is a decreasing/increasing function of $\gamma$ in the feasible region if the stationary point is not in the feasible region and that again makes one of the extreme points the maximizer. Therefore, for firm O to set a non-zero ratio the resulting profit should be larger than its profit when $\gamma = 0$ (i.e., $\frac{6}{289}(4 - k + 2M)^2 \leq \frac{6(k-2M+4w\gamma-4)}{(17-3\gamma)^2} \iff \frac{8-2k+4M}{3\gamma^2-9\gamma} \leq w \leq \frac{1}{68}(12 - 3k + 6M)$. Now that we know the best response allocation ratio $\gamma^*$ given any wholesale price $w$, we can show that firm C’s capacity has to be between $0 \leq k \leq 4 + 2M$ in order to have non-negative prices and profits in case 2.

Intuitively, depending on firm O’s decision on $\gamma$ at certain capacity levels firm C might either have binding or non-binding capacity. Hence, if firm C wants to use all of its capacity by getting
orders from firm O (i.e., $\gamma^* > 0$), it has to make sure that firm O prefers $\gamma^* > 0$ in binding capacity case to $\gamma^* = 0$ in non-binding capacity case in the overlapping capacity range (i.e.,

$$\frac{3}{50} (2 + M)^2 \leq \frac{6(-4k-2M+4w\gamma)^2}{(17-3\gamma)^2} \iff \frac{3(2+M)}{10} < k \leq \frac{1}{10} (2 + M)(11 + 3\gamma) \text{ and } \frac{8-2k+4M}{3\gamma^2-9\gamma} \leq w \leq \frac{3(2+M)(1+\gamma)-10k}{40\gamma}.$$  

This can be interpreted as the incentive compatibility condition of firm O to assign a non-zero allocation ratio to firm C in binding capacity case. Also, considering the wholesale price range for firm O to set non-zero $\gamma$ in binding capacity case, we conclude that if the capacity and the wholesale price are in the range $\tilde{A}$ (that is specified below), firm O will set the highest possible (extreme point) allocation ratio ($0 < \gamma^* \leq 1$).

$$\tilde{A} = \left\{ \begin{array}{ll} 0 \leq k \leq \frac{3(2+M)}{10} & \cup \quad \frac{8-2k+4M}{3\gamma^2-9\gamma} \leq w \leq \frac{1}{68} (12 - 3k + 6M) \\ \frac{3(2+M)}{10} < k \leq \frac{1}{10} (2 + M)(11 + 3\gamma) & \cup \quad \frac{8-2k+4M}{3\gamma^2-9\gamma} \leq w \leq \frac{3(2+M)(1+\gamma)-10k}{40\gamma} \end{array} \right.$$

3 - Stage 1 of the game

**Case 1** - Firm C’s profit function is strictly concave $(\frac{d^2 \pi_C}{dw^2} > 0)$. However, the critical point $w$ ($w = \frac{217 (2+M)}{876\gamma} > 0$) is not in the feasible range $(\frac{-14 - 7M}{19\gamma} \leq w \leq 0)$ for firm O to set a non-zero allocation ratio. So, if firm C wants to get orders from firm O, it will set the wholesale price to zero which results in $\pi_C = \frac{3}{50} (2 + M)^2$. We assume that firms choose cooperation if they get same profit outcomes compared to competition. As a result, depending on the capacity availability condition (1) (i.e., $\frac{3(2+M)}{10} \big|_{w=0,\gamma=1} < k$) firm C can always get a profit of $\pi_C = \frac{3}{50} (2 + M)^2$ from case 1.
Case 2 - Firm C’s profit function is strictly concave \( \frac{d^2 \pi_C}{d w^2} > 0 \). However, considering the acceptable wholesale price for firm O and feasible capacity range specified in \( \bar{A} \), we show that the optimal wholesale price \( w = \frac{17(2+M)+8(2-6k+M)\gamma+15(2+M)\gamma^2}{4\gamma(17-6\gamma+9\gamma^2)} \) is not in the acceptable range and is larger than the right hand side (cf. condition \( \bar{A} \)). Thus, firm C’s best wholesale price choices are: if \( 0 \leq k \leq \frac{3(2+M)}{10} \) then \( w = \frac{1}{68} (12 - 3k + 6M) \) OR if \( \frac{3(2+M)}{10} < k \leq \frac{1}{10} (2 + M)(11 + 3\gamma) \) then \( w = \frac{3(2+M)(1+\gamma)-10k}{40\gamma} \).

Now that we set the wholesale prices we can evaluate the actual \( \gamma^* \). Remember, firm O would choose the largest possible \( \gamma^* \) (closest to 1) when wholesale price makes it more beneficial for firm O to cooperate rather than only compete. Thus, in case 2: if \( w = \frac{1}{68} (12 - 3k + 6M) \) → \( \gamma^* = \frac{17k}{12-3k+6M} \) and if \( w = \frac{3(2+M)(1+\gamma)-10k}{40\gamma} \) → \( \gamma^* = 1 \). Note that, when capacity is small (i.e., \( 0 \leq k \leq \frac{3(2+M)}{10} \)) the optimal allocation ratio \( \gamma^* \) is smaller than 1 because firm C does not have enough capacity to produce all firm O’s demand.

Proof of Lemma 1

We define \( \bar{w} \) as the maximum acceptable wholesale price by firm O. We showed that in non-binding capacity case firm C will only choose \( w^* = 0 \) to get orders from firm O that changes condition (1) to \( \frac{3(2+M)}{5} < k \). In binding capacity case, knowing the \( \gamma^* \), condition \( \bar{A} \) can be written as:

\[
\bar{A} = \begin{cases} 
0 \leq k \leq \frac{3(2 + M)}{10} & \text{U} & - \frac{3(k - 2(M + 2))^3}{17k(13k - 9(M + 2))} \leq w \leq \frac{1}{68} (12 - 3k + 6M) \\
\frac{3(2 + M)}{10} < k \leq \frac{7(2 + M)}{5} & \text{U} & \frac{2k - 4M - 8}{6} \leq w \leq \frac{1}{20} (6 - 5k + 3M)
\end{cases}
\]
We already made sure that firm O prefers the binding capacity condition $\bar{A}$ over the non-binding capacity case. Though, in order for this to be subgame perfect, we also need to evaluate firm C’s outcome in the overlapping region. Thus, we have to compare firm C’s profit in binding capacity case when $w = \frac{1}{20}(6 - 5k + 3M)$ and $\frac{3(2+M)}{10} < k \leq \frac{7(2+M)}{5}$ with its profit in non-binding case (i.e., $\pi_C = \frac{2}{50}(2 + M)^2$). We find that beyond $\frac{3(2+M)}{5} < k$ firm C would always prefer to be in non-binding case. Therefore, knowing that firm C would deviate from binding capacity equilibrium, firm O only accepts $w \leq 0$ when $\frac{3(2+M)}{5} \leq k$. Consequently, using the capacity thresholds defined in the main text, we obtain $\bar{w}$.

**Proof of Proposition 1: The Equilibrium**

Firm C is the first mover and can choose to have $\gamma^* > 0$ or $\gamma^* = 0$ by its choice of wholesale price. This means that, firm C’s profit when $\gamma^* > 0$ should be at least equal to its profit when $\gamma^* = 0$, in both cases for the overlapping capacity range, so that it sets a wholesale price that is acceptable by firm O. Consequently, in order to find the equilibrium we compare firm C’s profit when $\gamma^* > 0$ with its profit from non-binding case and binding case when $\gamma^* = 0$. For instance, when capacity is large (i.e., $k > k_{34}$), we showed that, firm C prefers non-binding capacity case and because it has enough capacity to produce for both firms it will set $w^* = 0$. This wholesale price is acceptable for firm O; therefore, firm O will set $\gamma^* = 1$. Thus, both firms will assign optimal prices form non-binding capacity case even for overlapping capacity region. We use the same logic to derive the equilibrium conditions for each capacity level. Below we summarize the equilibrium conditions and decisions into four regions:
I. $k \leq k_{12}$ and $0 < M < \frac{31}{7}$ OR $\frac{(14M-6)}{7} < k \leq k_{12}$ and $\frac{3}{7} \leq M < \frac{6}{7} \rightarrow \gamma^* = 0$ (Binding Capacity Case);

II. $k_{12} < k < k_{23}$ and $0 < M < \frac{14}{23}$ OR $k_{12} < k < 2M$ and $\frac{14}{23} \leq M < \frac{6}{7} \rightarrow \gamma^* = 0$ (Non-Binding Capacity Case);

III. $k_{23} \leq k \leq k_{34}$ and $0 < M < \frac{14}{23}$ OR $2M < k \leq k_{34}$ and $\frac{14}{23} \leq M < \frac{6}{7} \rightarrow w^* = \frac{1}{20}(6 - 5k + 3M), \gamma^* = 1$ (Binding Capacity Case);

IV. $k > k_{34}$ and $0 < M < \frac{6}{7} \rightarrow w^* = 0$ and $\gamma^* = 1$ (Non-Binding Capacity Case).

**Firm O as the monopoly in the market**

Before claiming the above regions to be the equilibrium of the game we also consider a case where firm O buys out firm C’s capacity in order to be a Monopoly: $D_O = 2(1 - p_O); \pi_O = (p_O - w)\gamma D_O + p_O(1 - \gamma)D_O; \pi_C = w\gamma D_O$. Firm O’s profit function is strictly concave (i.e., $\frac{d^2 \pi_O}{dp_O^2} < 0$); therefore, $p_O^* = \frac{1}{2}(1 + w\gamma)$. Next, firm O decides on the allocation ratio that buys all firm C’s capacity ($\gamma D_O = k \rightarrow \gamma = \frac{1 - \sqrt{1 - 4kw}}{2w}$). Considering the participation constraint of both firms and the non-negativity of firm O’s market price we find the feasible regions that firm O can be the monopoly in the market: $0 < k \leq \frac{1}{2} \cup 0 < w \leq \frac{1}{4k}$ OR $\frac{1}{2} < k < 1 \cup 0 < w \leq 1 - k$. Firm C’s profit function is an increasing function in the wholesale price and thus will choose the largest feasible wholesale price (e.g., $w = \frac{1}{4k}$). The outcome for the hypothetical monopoly is: if $0 < k \leq \frac{1}{2}$, $\pi_C = \frac{1}{4}$ and $\pi_O = \frac{1}{8}$; if $\frac{1}{2} < k < 1$, $\pi_C = k(1 - k)$ and $\pi_O = \frac{1}{8}$.

---

1 The range for $M$ is derived from the necessary conditions to have competitive market.
\[ \frac{1}{8} \left( 1 + \sqrt{(1 - 2k)^2} \right)^2. \] Comparing the monopoly outcomes with two firms as competitors’ outcome we conclude that monopoly case can never be the equilibrium solution. Consequently, we claim that the 4 region equilibrium conditions presented above is the unique equilibrium of this sequential game (Proposition 1 is a different representation of the above mentioned equilibrium regions). Note that, in “Figure 2” and “Proposition 1” for illustrative and direct comparison reasons we only show the equilibrium until the minimum feasible upper bound of the \( M \) for all regions (i.e., \( M < \frac{3}{2} \)). Table A presents the optimal prices, demands and profits of the firms in each region of the equilibrium.

Proof of proposition 2: Sensitivity Analysis

a. From table A we can replace the corresponding demands: \( D_c^*(k_{23}^-) > D_c^*(k_{23}^+) \rightarrow \frac{3(2+M)}{10} > \frac{(10k-3M-6)}{10} \). This inequality is always true as long as \( k < k_{34} \) which is always larger than \( k_{23} \). Since firm O’s demand does not change around \( k_{23} \), the second expression (i.e., \( D_o^*(k_{23}^-) + D_c^*(k_{23}^-) > D_o^*(k_{23}^+) + D_c^*(k_{23}^+) \)) is the immediate result of the previous one.

b. This result is immediate from Table A as long as \( k < k_{34} \) which is always larger than \( k_{23} \).

c. Note that, profit function of firm C in region I and III is a concave function in its capacity where \( k_1 \) and \( k_3 \) are the maximizers of its profit in these regions respectively. These maximum points are situated within the regions (e.g., \( k_{23} \leq k_3 \leq k_{34} \)). Therefore, firm C’s profit will decrease beyond these points: \( if \ k_3 < k < k_{34} \rightarrow \frac{d\pi_c}{dk} = \frac{1}{10} (16 - 15k + 8M) < 0; \ if \ k_1 < k < k_{12} \rightarrow \frac{d\pi_c}{dk} = \frac{1}{17} (7(2 + M) - 24k) < 0. \) Moreover, we show that the absolute value of the slope
in region III is larger than that of region I: 
\[
\left. \frac{\partial \pi_C^*}{\partial k} \right|_{k=k_1+\Delta} < \left. \frac{\partial \pi_C^*}{\partial k} \right|_{k=k_3+\Delta} \rightarrow 7\left(2+M\right) - 24k
\]
\[
\left. \frac{1}{10} \left(16-15k + 8M\right) \right|_{k=k_3+\Delta}.
\]

**Proof of proposition 3: Sensitivity Analysis**

This proposition is an immediate result of Table A.

**Derivation of equilibrium when there is no competition in component market**

In this section we investigate the equilibrium conditions of the scenario presented in “Value of Competition in Component Market” section where firm C is the proprietary component manufacturer. We use the same logic as the main model to derive the equilibrium. The optimal prices are: Case 1) \( p_o^* = \frac{1}{35} (14 + 7M + 19w) \), \( p_c^* = \frac{1}{35} (14 + 7M + 9w) \) if and only if condition (2): \( \frac{1}{5} (6 + 3M - 4w) < k \) holds. Case 2) \( p_o^* = \frac{1}{7} (4 - k + 2M + 3w) \), \( p_c^* = \frac{1}{7} (10 - 6k + 5M - 3w) \).

Knowing the best response optimal prices firm C will choose the wholesale price: **Case 1** - Firm C’s profit function is strictly concave \( \left( \frac{d^2 \pi_C}{dw^2} > 0 \right) \) and has a maximum \( w = \frac{217(2+M)}{876} \) at \( \frac{d \pi_C}{dw} = 0 \). **Case 2** - Firm C’s profit function is strictly concave \( \left( \frac{d^2 \pi_C}{dw^2} > 0 \right) \) and has a maximum \( w = \frac{1}{10} (10 - 6k + 5M) \) at \( \frac{d \pi_C}{dw} = 0 \). Note that, in binding case firm C’s capacity has to be small enough so that firm C can use all of its production limit while maintaining positive prices and profits (i.e., \( 0 \leq k \leq \frac{1}{6} (10 + 5M) \)).

**Overlap between case 1 & case 2**

Similar to the main model, there is an overlap between the capacity conditions. Since firm C is the first mover, considering the incentive compatibility conditions of firm O, it decides which of
the optimal wholesale prices to choose when capacity is in the overlapping region. Consequently, firm C’s optimal action in each region is: **A) Binding Capacity Case:** \[ k \leq \frac{5}{6} + \frac{5M}{12} + \]
\[
\frac{1}{12} \sqrt{\frac{5}{73}} (2 + M) \rightarrow w = \frac{1}{10} (10 - 6k + 5M); \]

**B) Non-Binding Capacity Case:** \[ k > \frac{5}{6} + \frac{5M}{12} + \]
\[
\frac{1}{12} \sqrt{\frac{5}{73}} (2 + M) \rightarrow w = \frac{217(2+M)}{876}. \]

**Firm C as the monopoly in the market**
Before claiming the above regions to be the equilibrium of the game we also consider a case where firm C chooses to be a Monopoly. In this scenario, firm C faces a capacity constraint (i.e., \( D_c < k \)) and only decides on its market price. Solving for the optimal price considering the price and demand non-negativity, we have: if \( k \geq 1 \rightarrow p_c = \frac{1}{2} \rightarrow \pi_c = \frac{1}{2} \) **OR if** \( k < 1 \rightarrow p_c = \frac{1}{2} + \)
\[
\frac{1}{2} \sqrt{1 - 2k + k^2} \rightarrow \pi_c = \frac{1}{2} (2k - k^2). \]

Firm O is not a strategic player in this scenario. Comparing with the cooperation scenario, we show that there are some cases that firm C prefers to be monopoly in the final product market and thus results in four different regions of equilibrium depending on firm C’s capacity and product substitutability \( M \):

**A. Binding Capacity Case**
1. Coopetition: if \( k \leq \frac{1}{876} (365 + \sqrt{365})(2 + M) \) and \( M \geq k \)

2. Monopoly:
   \[
   \begin{cases}
   if \ 0.9612 < k \leq 1 \ and \ M \leq \frac{1}{91} (\sqrt{39858} \sqrt{(2-k)k} - 182) \\
   if \ k \leq 0.9612 \ and \ M < \frac{k}{5}
   \end{cases}
   \]

**B. Non-Binding Capacity Case**
3. Coopetition:
   \[
   \begin{cases}
   if \ \frac{1}{876} (365 + \sqrt{365})(2 + M) < k < 1 \ and \ M \geq \frac{1}{91} (\sqrt{39858} \sqrt{(2-k)k} - 182) \\
   \end{cases}
   \]

4. Monopoly: \( if \ k \geq 1 \ and \ M \leq 0.1939 \)
**Firm C as component supplier**

Finally, we consider a case where firm C chooses to be only the component provider for firm O. In this case firm C chooses the wholesale price $w$ and only after that firm O will decide on the market price of its product. Firm O’s profit function is strictly concave ($\frac{d^2 \pi_O}{dp_o^2} > 0$). So, FOC gives: $p_O^* = \frac{1+w}{2}$. Then, firm C chooses its optimal wholesale price. Having a strictly concave profit function ($\frac{d^2 \pi_p}{dw^2} > 0$): if $k \geq \frac{1}{2} \rightarrow w = \frac{1}{2} \rightarrow \pi_C = \frac{1}{4} OR if k < \frac{1}{2} \rightarrow w = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4(k^2 - k)} \rightarrow \pi_C = k - k^2$. Firm C’s profit when only a component provider is always dominated by its profit in the monopoly or coopetition case presented in the 4 region equilibrium above. Consequently, we claim that the 4 region equilibrium conditions is the unique equilibrium of this sequential game. Table B presents the optimal prices, demands and profits of the firms in each region of the equilibrium.

**Proof of proposition 4: Value of competition**

In this proposition we evaluate the effect of capacity and product substitutability on value of competition. From Table A and B we can find $V_O$ for any given capacity level and product substitutability $M$ (e.g., Non - binding cooperation: $V_O = \left[ \frac{3}{50} (2 + M)^2 \right] - \left[ \frac{361}{31974} (2 + M)^2 \right] = \frac{19468(2+M)^2}{399675}$). There are in total 11 different values of competition outcomes for firm O depending on capacity level and product substitutability. Knowing the $V_O(k, M)$ functions we can derive the results in proposition 4. The results for firm C can be driven in the same way.
Table A: Equilibrium profits, prices and demands of the firms.

<table>
<thead>
<tr>
<th>Equilibrium Region</th>
<th>Firm C</th>
<th>Firm O</th>
</tr>
</thead>
<tbody>
<tr>
<td>I) $k \leq k_{12}$</td>
<td>$\pi_c = \frac{k(7(2 + M) - 12k)}{17}$</td>
<td>$\pi_o = \frac{6}{289}(4 - k + 2M)^2$</td>
</tr>
<tr>
<td></td>
<td>$p_c = \frac{(7(2 + M) - 12k)}{17}$</td>
<td>$p_o = \frac{2}{17}(4 - k + 2M)$</td>
</tr>
<tr>
<td></td>
<td>$D_c = k$</td>
<td>$D_o = \frac{3}{17}(4 - k + 2M)$</td>
</tr>
<tr>
<td>II) $k_{12} &lt; k &lt; k_{23}$</td>
<td>$\pi_c = \frac{3}{50}(2 + M)^2$</td>
<td>$\pi_o = \frac{3}{50}(2 + M)^2$</td>
</tr>
<tr>
<td></td>
<td>$p_c = \frac{2 + M}{5}$</td>
<td>$p_o = \frac{2 + M}{5}$</td>
</tr>
<tr>
<td></td>
<td>$D_c = \frac{3(2 + M)}{10}$</td>
<td>$D_o = \frac{3(2 + M)}{10}$</td>
</tr>
<tr>
<td>III) $k_{23} \leq k \leq k_{34}$</td>
<td>$\pi_c = \frac{16k(2 + M) - 3(2 + M)^2 - 15k^2}{20}$</td>
<td>$\pi_o = \frac{3}{50}(2 + M)^2$</td>
</tr>
<tr>
<td></td>
<td>$p_c = \frac{(26 - 15k + 13M)}{20}$</td>
<td>$p_o = \frac{(14 - 5k + 7M)}{20}$</td>
</tr>
<tr>
<td></td>
<td>$D_c = \frac{10k - 3M - 6}{10}$</td>
<td>$D_o = \frac{3(2 + M)}{10}$</td>
</tr>
<tr>
<td>IV) $k &gt; k_{34}$</td>
<td>$\pi_c = \frac{3}{50}(2 + M)^2$</td>
<td>$\pi_o = \frac{3}{50}(2 + M)^2$</td>
</tr>
<tr>
<td></td>
<td>$p_c = \frac{2 + M}{5}$</td>
<td>$p_o = \frac{2 + M}{5}$</td>
</tr>
<tr>
<td></td>
<td>$D_c = \frac{3(2 + M)}{10}$</td>
<td>$D_o = \frac{3(2 + M)}{10}$</td>
</tr>
</tbody>
</table>
**Table B:** Equilibrium profits, prices and demands of the firms when there is no competition in component market.

<table>
<thead>
<tr>
<th>Equilibrium Region</th>
<th>Firm C</th>
<th>Firm O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>$\pi_C = \frac{1}{10}(10k - 6k^2 + 5kM)$</td>
<td>$\pi_O = \frac{3k^2}{50}$</td>
</tr>
<tr>
<td></td>
<td>$p_C = \frac{1}{10}(10 - 6k + 5M)$</td>
<td>$p_O = \frac{1}{10}(10 - 4k + 5M)$</td>
</tr>
<tr>
<td></td>
<td>$D_C = \frac{7k}{10}$</td>
<td>$D_O = \frac{3k}{10}$</td>
</tr>
<tr>
<td>2)</td>
<td>$\pi_C = \frac{1}{2}(2k - k^2)$</td>
<td>$\pi_O = 0$</td>
</tr>
<tr>
<td></td>
<td>$p_C = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 2k + k^2}$</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td>$D_C = k$</td>
<td>$-$</td>
</tr>
<tr>
<td>3)</td>
<td>$\pi_C = \frac{91}{876}(2 + M)^2$</td>
<td>$\pi_O = \frac{361}{31974}(2 + M)^2$</td>
</tr>
<tr>
<td></td>
<td>$p_C = \frac{77}{292}(2 + M)$</td>
<td>$p_O = \frac{293}{876}(2 + M)$</td>
</tr>
<tr>
<td></td>
<td>$D_C = \frac{119}{438}(2 + M)$</td>
<td>$D_O = \frac{19}{146}(2 + M)$</td>
</tr>
<tr>
<td>4)</td>
<td>$\pi_C = \frac{1}{2}$</td>
<td>$\pi_O = 0$</td>
</tr>
<tr>
<td></td>
<td>$p_C = \frac{1}{2}$</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td>$D_C = 1$</td>
<td>$-$</td>
</tr>
</tbody>
</table>