Optimal Management of Wind Energy with Storage: Structural Implications for Policy and Market Design

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Abstract: It is well known that the generation resource uncertainty induced by significant wind capacity raises concerns about grid security, price stability, and revenue adequacy. One of the most promising solutions is the use of utility-scale energy storage, although the question of general implementation of this strategy remains unanswered. This paper uses a simplified model to show that simple rules exist that govern the decision to generate or store energy from a hybrid wind-storage system. The heuristics developed consider the combination of storage efficiency, electricity price, and shortfall penalty and wind forecast characteristics to guide the decision of whether to bid energy into the electricity market or not. Specifically, this paper develops the optimal strategy for use of a simplified system of an energy storage unit with a wind generator. The solution is analyzed using a dynamic programming formulation in a simplified framework over a multiperiod planning horizon. The analysis of the solution under all regimes yields insightful structural solutions regarding the conditions under which the wind generator should bid into the energy market and when it should not. The results also provide insight into the specific implications of forecast accuracy and market design on the need for storage. This analysis allows additional conclusions to be drawn about the value of various storage technologies based on their capacity and efficiency characteristics. However, the most important contribution of this work is the understanding of the importance of market penalties in encouraging participants to either improve forecasting ability or, perhaps more realistically, contract storage to mitigate shortfall risk. Improving both forecasting accuracy and storage capabilities results in value reduction for both.
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Introduction

The primary goal for integration of renewable resources into power systems is the production of low-cost sustainable electricity to the world's population. There has been no shortage of research into the management of these intermittent sources, and the consensus indicates that in virtually any situation, wind power will require some sort of coupled resource to mitigate its variability (DeCesaro et al. 2009). Many resources have potential for this application, including, but not limited to, storage, responsive demand, and dedicated reserves. Note that these categories are broad, can be used in combination, and storage can take many forms from pumped hydro, to compressed air energy storage, to batteries and flywheels. While the body of literature discussing the implementation of such solutions is growing, there is no single answer for the best practice for optimal management of such systems.

The related research that has considered the coupling of wind and various types of storage can be roughly divided into two categories: the use of optimization to design the coupled system and the development of optimal control strategies.

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Studies of the first category use optimization methods to determine the most financially efficient storage capacity for a specific system. For example, one of the earliest studies by (Castronuovo and Lopes 2004b) considers the optimal sizing of pumped storage facilities to maximize revenue to the wind farm operator. A later work by Abbey and Joos (2009) uses a two-stage, scenario-based optimization to size the energy storage system (ESS) to minimize the expected cost of serving load. This model uses very detailed models of the financial parameters of the storage unit, and is solved in the numerical package General Algebraic Modeling System (GAMS). Most recently, Connolly et al. (2012) consider a case study wherein a one-year simulation on an hourly basis is used to consider various sizes of a pumped hydro storage facility in Ireland to be used in conjunction with high penetrations of wind. The authors find that the use of pumped hydro storage has significant potential in the mitigation of wind variability, but the economics have yet to be become compelling.

The second category of research into the use of storage to mitigate wind variability addresses the development of optimal control policies to direct operational decisions. One of the first forays into this area was presented in Castronuovo and Lopes (2004a), which considers a wind-hydro coordinated energy system as a linear program with stochasticity introduced through wind scenarios. The Monte Carlo simulation framework allows the consideration of a range of possible wind outcomes, providing a boundary on power available from the system. In later work, these authors consider the use of probabilistic wind forecasts in three different operating approaches to assess revenue differences through the coordination of two wind farms with a large pumped storage facility (Castronuovo et al. 2013). Garcia-Gonzalez et al. (2008) consider the use of pumped hydro to mitigate wind farm output fluctuations through a two-stage stochastic program. The first-stage decisions are optimal bids in the day-ahead market, with the option in the second stage to update operations accounting for wind output realization. The formulation is implemented in the GAMS software package for a sample system to illustrate the financial value of coordinated operations between the wind and pumped hydro units. Denault et al. (2009) investigate the way in which wind power can be used in conjunction with a large hydro reservoir system to mitigate shortfall risk due to drought conditions. Xie et al. (2012) develop a model predictive control approach for managing a wind farm with battery storage, using a moving horizon forecast to show the financial benefit to both the storage system and the wind farm operators. Kim and Powell (2011) develop a closed-form model for determining optimal energy commitments in the spot market based on distributions for exogenous spot price, wind forecast errors, and storage levels. The model in this paper is perhaps most closely related to the approach taken by Kim and Powell, whose continuous time model contains considerably more detail than our discrete time model, and which admits closed-form solutions in the infinite time horizon case. The fact that this model is much more stylized allows for obtaining interesting closed-form results for finite time horizons (in the "medium penalty" case described in this study).

This paper considers the market penalty for wind generator shortfalls as an endogenous variable that can be manipulated to encourage system-beneficial behavior in market participants providing wind energy. A stochastic dynamic programming approach is also used here, for a stylized model of the wind energy and the storage facility. Model analysis shows the importance of the interplay between forecasting ability, storage efficiency, and market penalty that cannot be ignored if wind generators are to contribute to energy markets in sustainable and beneficial manner.

The remainder of this paper is organized as follows: in section 2, the dynamic programming approach to the optimal control problem will be described. In section 3, the model is analyzed, followed by a description of valuation of storage and forecasting accuracy in section 4. Discussion and conclusions are provided in section 5.

Dynamic Programming Approach to Optimal Control Problem

This section develops a stylized model of the decisions faced by the operator of a joint wind/storage facility comprising a wind turbine and a small battery storage to provide a basis for determining the optimal control strategy. In each period, the wind turbine is assumed to operate either at full capacity, generating energy with value M, or not at all. The periods are indexed by k, counting from the end of the planning horizon. The probability of there being sufficient wind to run the wind turbine in the kth period is denoted by p_k.

The assumption is that the battery is large enough to hold exactly the energy output generated by the wind turbine during one period.

Before each period, a wind producer must decide whether to bid electricity; if a decision to bid is made and the wind is blowing the producer receives M, the price of electricity. On the other hand, if the decision is to bid and the wind is not blowing, the producer faces a penalty of xM. This simplified price model is designed to capture either a situation where wind producers are paid a constant high feed in tariff or in which a long-term contract is negotiated between a wind producer and another party. If time-varying power prices are available, either due to a time-of-use pricing schedule or arising from a power market, a storage facility can obtain significant additional value by engaging in arbitrage activities.

To increase the complexity of this problem slightly, assume that the wind producer has access to one unit of battery storage.

Table 1. Benefit Matrix: Empty Battery

State	Bid	Do not bid
Wind	M	$(1-\gamma)M$
No wind	$-xM$	0

Table 2. Benefit Matrix: Full Battery

State	Bid	Do not bid
Wind	$M + (1-\gamma)M$	$(1-\gamma)M$
No wind	$(1-\gamma)M$	$(1-\gamma)M$

At a given time period, the storage is either empty or full. The wind producer pays γM for using the unit, but it can be used to help avoid the potentially hefty penalty imposed in a case where a bid decision has been made, but the wind doesn't blow. Tables 1 and 2 show the benefit matrices associated with an initially empty battery, as well as an initially full one, assuming a single period.

Let B denote the decision to bid, and N denote the decision not to bid. E represents an empty battery, whereas F represents a full battery. So $V(F,B,k)$ describes the expected value of the wind/storage system with k periods remaining, a full battery, and a firm offer to supply one unit of power in the next time period; $V(E,N,k)$ describes the value of an empty facility with k periods remaining and no offer of power, etc. Note that all such values assume optimal operation (given the information possessed by the operator at the time of bidding) in the future.

Assume further that at the end of the planning period, a full storage facility is assigned a value of $(1-\gamma)M$ and an empty facility a value of zero:

$$V(E,0) = 0 \qquad V(F,0) = (1-\gamma)M$$

Note that in this case, these "initial" conditions are really terminal conditions; in other words, this is a *backward* dynamic programming formulation. It is common in dynamic programming problems to begin with a time-reversal transformation in the formulation of the problem, by always referring to the number of periods remaining and by defining that as the time variable.

Given an empty battery with k periods remaining, the value of each decision after one time period can be written as

$$V(E,N,k) = p_k V(F,k-1) + (1-p_k)V(E,k-1)$$
$$V(E,B,k) = p_k M + (1-p_k)(-xM) + V(E,k-1)$$

In other words, if the decision is not to bid, the battery gets filled when the wind blows (a fraction p_k of the time), otherwise nothing happens ($1-p_k$ of the time). If the decision is to bid, M is received for the wind energy if it blows, but there is a penalty of $-xM$ if it is calm; in either event, the facility continues to be empty. Now, just $k-1$ periods remain, and so a value of $V(E,k-1)$ is retained. One will bid if more expected value comes from that action than from not bidding, so will bid when $V(E,B,k) > V(E,N,k)$. This maximization carries through to the next time step, so choose $V(E,k) = \max[V(E,N,k), V(E,B,k)]$, denoted by $V(E,k)$.

A similar set of equations can be developed for the full battery case, this time using $V(F,N,k)$ and $V(F,B,k)$. This can be written as

$$V(F,N,k) = p_k V(F,k-1) + (1-p_k)V(F,k-1) = V(F,k-1) \tag{1}$$

$$V(F,B,k) = p_k[M + V(F,k-1)] \tag{2}$$

$$(1-p_k)\max[-xM+V(F,k-1),(1-\gamma)M+V(E,k-1)] \quad (3)$$

If the decision is not to bid, then simply do nothing; if there is wind, it cannot be stored because the battery is already full. However we do receive $V(F,k-1)$, representing the value of the system given that it is full with $k-1$ periods remaining. If the decision is to bid, then it is not quite as simple. If there is wind, sell the energy for M and retain full facility with value $V(F,k-1)$. The max() expression within $V(F,B,k-1)$ indicates that there are two choices if there is no wind: pay a penalty and keep the battery full, or use the energy stored in the battery to make a delivery commitment. The purpose of a bid is to make the most money, so write $V(F,k)=\max[V(F,N,k),V(F,B,k)]$.

Collecting these results yields the following system of difference equations:

$$V(E,N,k)=p_kV(F,k-1)+(1-p_k)V(E,k-1) \quad (4a)$$

$$V(E,B,k)=p_k[M+V(E,k-1)] + (1-p_k)[-xM+V(E,k-1)] \quad (4b)$$

$$V(E,k)=\max[V(E,N,k),V(E,B,k)] \quad (4c)$$

$$V(F,N,k)=V(F,k-1) \quad (4d)$$

$$V(F,B,k)=p_k[M+V(F,k-1)]+(1-p_k)\max[(1-\gamma)M + V(E,k-1)-xM+V(F,k-1)] \quad (4e)$$

$$V(F,k)=\max[V(F,N,k),V(F,B,k)] \quad (4f)$$

with initial conditions

$$V(E,0)=0 \quad (4g)$$

$$V(F,0)=(1-\gamma)M \quad (4h)$$

Equipped with this model, the next step is to determine the optimal control strategy.

Model Analysis

Given the model system defined in the previous section, structural results can be obtained and analyzed to provide various insights by first examining the difference $V(F,k)-V(E,k)$. This difference is crucial in determining this strategy. In fact, the following theorem regarding this difference allows one, at least in the special case of constant p_k, to obtain the optimal operating strategy for any x:

$$V(F,k)-V(E,k)=\max[V(F,B,k),V(F,N,k)] - \max[V(E,B,k),V(E,N,k)] \quad (5a)$$

$$\begin{aligned}&=\max\{p_k[M+V(F,k-1)]\\&\quad+(1-p_k)\max[-xM+V(F,k-1),(1-\gamma)M\\&\quad+V(E,k-1)],V(F,k-1)\}-\max\{p_k[M+V(E,k-1)]\\&\quad+(1-p_k)[-xM+V(E,k-1)],p_kV(F,k-1)\\&\quad+(1-p_k)V(E,k-1)\}\end{aligned} \quad (5b)$$

Now define $n(k)=[V(F,k)-V(E,k)]/M$. After some simple manipulations, the result is

$$\begin{aligned}n(k)&=n(k-1)+\max\{-(1-p_k)x,-(1-p_k)\\&\quad\times[n(k-1)-(1-\gamma)],-p_k\}\\&\quad-\max[-x(1-p_k),p_kn(k-1)-p_k]\end{aligned} \quad (6a)$$

$$\begin{aligned}&=n(k-1)+\min[x(1-p_k),p_k-p_kn(k-1)]\\&\quad-\min\{x(1-p_k),(1-p_k)[n(k-1)-(1-\gamma)],p_k\}\end{aligned} \quad (6b)$$

Now let $m(k)=n(k)-(1-\gamma)$. With this, the following result is easily obtained:

$$\begin{aligned}m(k)&=m(k-1)+\min[x(1-p_k),p_k\gamma-p_km(k-1)]\\&\quad-\min[x(1-p_k),(1-p_k)m(k-1),p_k]\end{aligned}$$

Now examine the special case of constant p:$p_k=p$ for all k. In this special case, the system of difference equations [Eq. (6)] can be solved in closed form, which will yield many useful insights. With constant p, the difference equation for $m(k)$ has the solution

$$m(k)=\min[\gamma p,x(1-p^k)] \quad (7)$$

Solution of Constant p System

For the system of Eq. (6) with $p_k=p\,\forall k$

$$V(F,k)-V(E,k)=(1-\gamma)M+\min[p\gamma M,(1-p^k)xM], \quad \forall\ k\geq 0 \quad (8)$$

Proof. By induction, divide into two cases on $\boldsymbol{x}(1-\boldsymbol{p}^{\boldsymbol{k}})>\boldsymbol{\gamma p}$ or vice versa, and lots of manipulations of min and max functions. See Appendix I. Equipped with this theorem, the optimal control can be stated.

Optimal Control for Constant p

1. If the wind generator bids and the wind doesn't blow, it is always better to empty the storage than pay a nonnegative penalty, no matter how small.
2. Bidding is always optimal when the storage facility is full.
3. When empty, bid only when penalties are low relative to other problem parameters; otherwise, do not bid.

To be precise, when empty, it is better to bid if and only if

$$\gamma p>x(1-p^k) \quad (9)$$

All of these results are proved in Appendix II. The basic idea is sketched here for point 1. From (9) with constant p_k, it is better to drain the storage rather than pay the penalty if

$$-xM+V(F,k-1)\leq(1-\gamma)M+V(E,k-1) \quad \forall\ k \quad (10)$$

This may be cast in terms of $m(k)=[V(F,k)-V(E,k)]/M-(1-\gamma)$, in which form the result takes the simpler form of using the storage if $m(k-1)<x$. This result clearly holds, since $m(k)=\min[\gamma p,x(1-p^k)]$.

Write (10) to isolate x as follows: If

$$x<\gamma p/(1-p^k) \quad (11)$$

then bid when empty; otherwise, do not bid. Note that this decision threshold scales linearly with γ. The value in not bidding comes from the ability to fill the battery when the wind blows; this opportunity is less interesting when batteries are very lossy (large γ), so larger penalties are required to force battery use. The decision threshold also scales with p: for all $k = 1, 2, 3 \ldots p/(1 - p^k)$ is a increasing function of p. This also makes sense; the more likely the wind is to blow, the less risk taken by bidding, and the larger the penalties need to be to induce the use of storage. Now, divide x into three parameter regimes and examine each individually:

Small Penalties ($x \leq \gamma p$)
If $x \leq \gamma p$, then (11) is satisfied for all values of k. So it is always optimal to bid when empty for these very small penalties. In this case, the difference equations take a very simple linear form, and it is easy to show that

$$V(E, k) = kM[p - x(1 - p)] \tag{12}$$

$$V(F, k) = kM[p - x(1 - p)] + (1 - \gamma)M + xM(1 - p^k) \tag{13}$$

in both cases for all $k >= 0$.

Large Penalties [$x \geq \gamma p/(1 - p)$]
if $x \geq \gamma p/(1 - p)$, then $x \geq \gamma p/(1 - p^k)$ for all $k = 1, 2, 3, \ldots,$ and it is never optimal to bid when empty (the penalties are just too large). Here as well, the difference equations take a simple linear form, although different from their small-penalty counterparts. It is also easy to show that

$$V(E, k) = kpM[1 - \gamma(1 - p)](k \geq 0) \tag{14}$$

$$V(F, k) = M(kp + 1)[1 - \gamma(1 - p)] \tag{15}$$

where the latter relation only holds for $k \geq 1$; when $k = 0$, $V(F, 0) = M(1 - \gamma)$. It is interesting to note that the value functions do not depend on x at all here. This does make sense because the bidding structure is such that one never bids when there is any danger in having to pay a penalty, so penalties are never paid.

Medium Penalties [$\gamma p < x < \gamma p/(1 - p)$]
If $\gamma p < x < \gamma p/(1 - p)$, then the outcome of the inequality in (3.4) depends on k. For fixed γ and p, $\gamma p/(1 - p^k)$ is a decreasing function of k. So it is possible that, for small values of k, $x < \gamma p/(1 - p^k)$ but for larger values of k, the opposite is true. Now, choose k^* to be the largest value of k for which $x < \gamma p/(1 - p^k)$ holds. In other words, k^* is the largest integer less than z, where $x = \gamma p/(1 - p^z)$, implying $1 - \gamma p/x = p^z$. Since $x > \gamma p$, and since γp here is positive, logarithms of both sides obtain: $z \ln p = \ln(1 - \gamma p/x)$ or

$$z = \frac{\ln(x - \gamma p) - \ln x}{\ln p} \tag{16}$$

So for medium penalties, the strategy is for $k \leq k^*$, bid, but for $k > k^*$, do not bid, where k^* is the largest integer $\leq z$. It is fairly easy to work out the analytic solution for $V(E, k)$ and $V(F, k)$ in this case as well, by using the "always bid" solution for $k \leq k^*$ and then using the resulting $V(E, k^*)$ and $V(F, k^*)$ as new "initial conditions" for a new "never bid" difference equation:

$$V(E, k) = kM[p - x(1 - p)], \qquad k < k^* \tag{17}$$

and

$$V(F, k) = kM[p - x(1 - p)] + (1 - \gamma)M + xM(1 - p^k), \quad k < k^* \tag{18}$$

when $k > k^*$ we can use the initial condition $V(E, k^*) = k^* M[p - x(1 - p)]$ and write

$$\begin{aligned} V(E, k) &= k^* M[p - x(1 - p)] + (k - k^*)pM[1 - \gamma(1 - p)] \\ &= kpM - k^* Mx(1 - p) - (k - k^*)M\gamma p(1 - p), \quad k \geq k^* \end{aligned}$$

Hence

$$\begin{aligned} V(F, k) &= V(E, k) + M[1 - \gamma(1 - p)] \\ &= kpM - k^* Mx(1 - p) - (k - k^*)M\gamma p(1 - p) \\ &\quad + M[1 - \gamma(1 - p)] \end{aligned} \tag{19}$$

Optimal Bidding Behavior with Time-Varying Probabilities

The model developed here also is generally extensible to time-varying probabilities of wind through a straightforward numerical implementation of Eqs. (2.1)–(2.8). As an illustration, this study uses deterministic diurnal wind probabilities from the northeastern United States, estimated from the NREL Eastern Wind Integration Study data set (Brower 2009) to consider the impact on optimal strategy. Fig. 1 illustrates the change in the optimal bidding strategy for various penalty values with a diurnal pattern of wind probabilities.

Values of Storage and Forecast Accuracy

This section uses the previous model and optimal control strategy to assess the value of storage to the wind producer, as well as the value of "perfect" forecasts for wind producers with and without storage. The authors were able to show that, under simplified conditions, it is always optimal to bid when the storage facility is full. Moreover, when penalties are small, it is optimal to bid when the storage facility is empty and when penalties are large, it is optimal not to bid when the storage facility is empty. When penalties are moderate, it is optimal not to bid far from maturity and optimal to bid closer to maturity. The point at which the decision switch occurs can be determined.

Value of Storage under Constant Wind Probability

Now that it is clear how to operate a storage facility if one were provided, how much value is being added by the storage? Consider a simple wind turbine with no storage unit; the value of the turbine with k periods remaining is

$$\begin{aligned} W(B, k) &= p[M + W(k - 1)] + (1 - p)[-xM + W(k - 1)] \\ &= W(k - 1) + [p - x(1 - p)]M \\ W(N, k) &= pW(k - 1) + (1 - p)W(k - 1) \\ &= W(k - 1) \\ W(k) &= \max[W(B, k), W(N, k)] \\ &= W(k - 1) + M \max[p - x(1 - p), 0] \end{aligned}$$

$$W(k) = kM \max[p - x(1 - p), 0] \tag{20}$$

since $W(0) = 0$. From this, it is clear that if $x < p/(1 - p)$, then the optimal strategy is to always bid, yielding $W(k) = kM[p - x(1 - p)]$. If $x > p/(1 - p)$, then it is optimal to never bid, with $W(k) = 0$. In other words, large penalties destroy the value of wind in the absence of storage. Looking back at the solution of the difference equation when penalties are large, the authors are able to state that for a k period storage facility, the added value from storage is

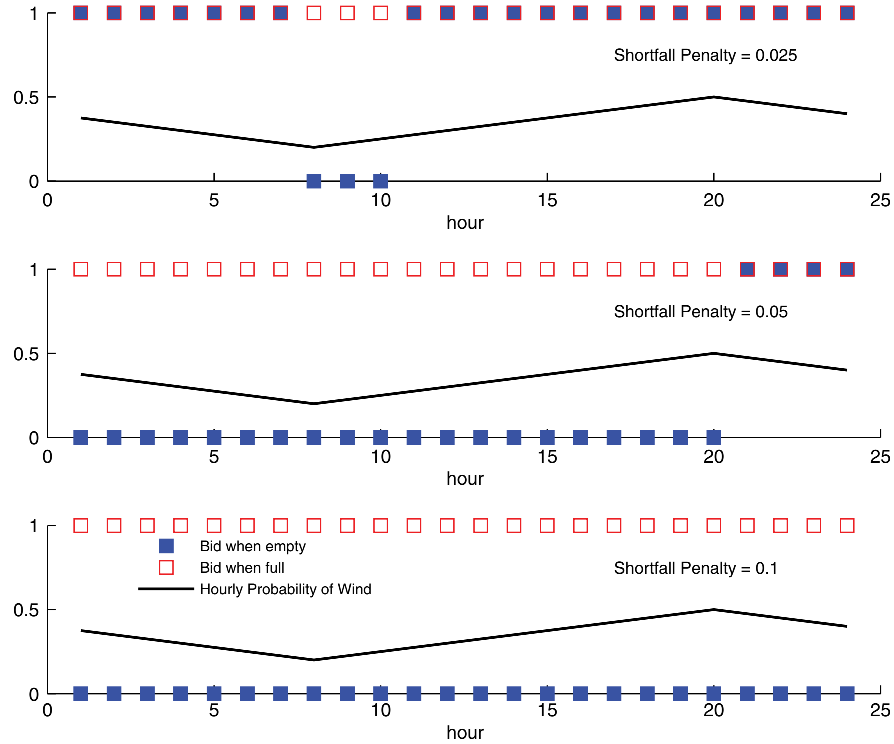

Fig. 1. The decision to bid or not bid changes with shortfall penalty: With lowest penalty ($x = 0.025$), the wind generator will bid regardless of whether stored energy is available, except with lowest probability of wind in hours 8 through 10; when the penalty function is high ($x = 0.1$), the generator does not bid when storage is empty (dark filled squares remain at 0)

$$kpM[1 - \gamma(1 - p)]$$

under the assumption that the storage is initially empty, and

$$(k + 1)pM[1 - \gamma(1 - p)]$$

under the assumption that the storage is initially full. Both equations imply that more wind is better and a more efficient (smaller γ) storage facility is better. A storage facility that will last longer is also better; this especially makes sense if the purchasing cost of such a facility is considered.

With numerical estimates for assumptions about wind and battery efficiency, we can consider the relative value of battery storage in each regime for x. A static *probability of wind*, though highly stylized, is useful for considering the impact of market penalties and can be estimated from historical data, for example (Wind Energy Center 2009; U.S. National Renewable Energy Laboratory 2010).

Also, γ can be estimated using a sodium sulfur battery as an example (Baker 2008; Rydh and Sandén 2005a, b; Dufo-Lóez et al. 2009). Use a round-trip efficiency of $\beta = 80\%$, or in other words, a fractional loss due to storage of $\gamma = 0.2$. We also choose an average power price of \$0.04/kWh. The only parameter that an estimate cannot be obtained for based on past data is x. Therefore, explore the effect of x on the value of the wind-storage system.

When penalties are large, the value is unaffected by x; this is because the optimal strategy avoids these penalties by not bidding when the storage facility is empty. Fig. 2 shows $V(F, k)$ and $V(E, k)$ as the number of periods remaining, k, increases, with $x = 0.15$. It is important to note that the result is the same for all $x > 0.128$ since this is the point at which penalty avoidance is complete and the generator will never bid when the storage unit is empty. As expected, $V(F, k) > V(E, k) \forall\ k$. When penalties are small, the value is affected by x. This is because the optimal strategy is always to bid, and hence the penalty must be paid on calm days when the storage facility is empty. Fig. 3 shows $V(F, k)$ and $V(E, k)$, $k = 100$, for $0 \leq x \leq p\gamma = 0.078$. This figure shows that as the penalty level increases, $V(E, k)$ and $V(F, k)$ decrease. As expected, $V(F, k) > V(E, k) \forall\ k$. Finally, for medium penalties, consider $k > k^*$. As k^* is dependent on x, it is assumed that $k = k^* + 10$. For the parameter set considered, medium penalties include $0.078 < x < 0.6393$, as shown in Fig. 4. When penalties are near $p\gamma = 0.078$, $V(E, k)$, and $V(F, k)$ decrease more rapidly,

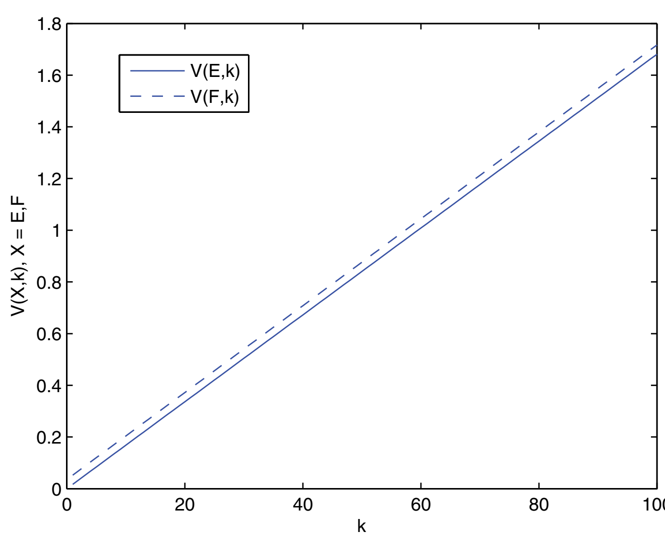

Fig. 2. $V(F, k)$ and $V(E, k)$ as k increases: $p = 0.39$, $\gamma = 0.2$, $M = 0.04$

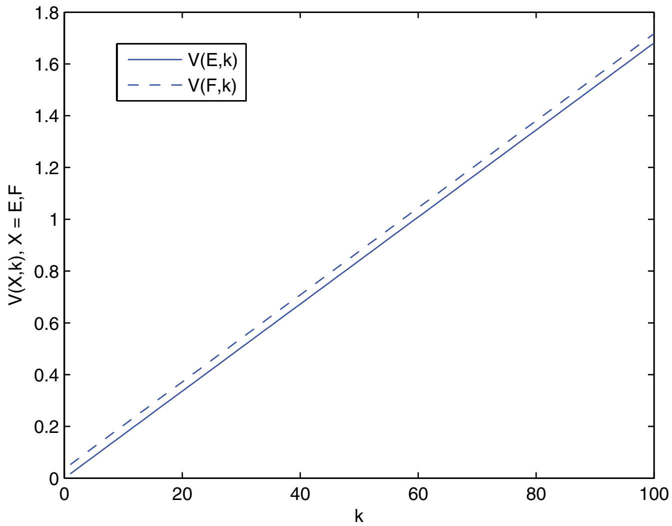

Fig. 3. $V(F,k)$ and $V(E,k)$ for $0 \leq x \leq p\gamma$: 0.1 cm $p = 0.39$, $\gamma = 0.2$, $M = 0.04$, $k = 100$

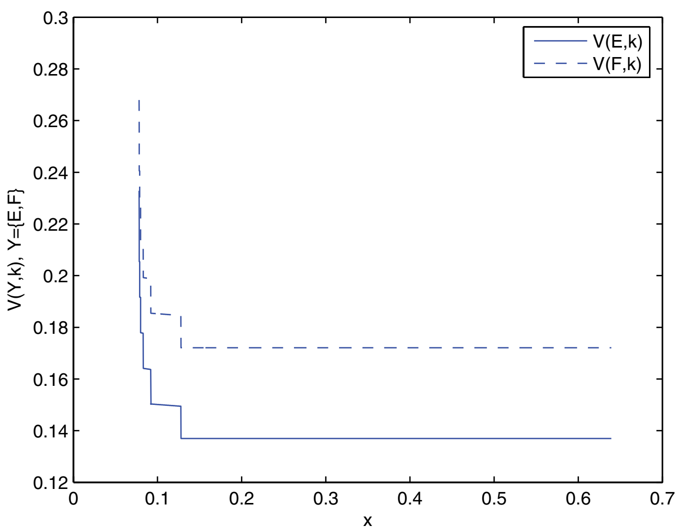

Fig. 4. $V(F,k)$ and $V(E,k)$ for $p\gamma \leq x \leq \gamma p/(1-p)$: $p = 0.39$, $\gamma = 0.2$, $M = 0.04$

in contrast to penalties near $\gamma p/(1-p)$. In fact, as penalty levels approach $\gamma p/(1-p)$, values are no longer affected by the level of penalty.

The results show that the "never bid when empty" condition, $x(1-p) > \gamma p$, holds for penalty rates greater than about $x = 0.13$. In other words, severe nondelivery penalties are not required to induce the use of storage. However a realistic wind storage model will be much more detailed than the simple 4 parameter model outlined in this paper.

Perfect Forecasts

It is very easy to value the wind farm with a perfect one-period forecast. In that case, always bid when there is going to be wind and, provided any positive penalty rate $x > 0$, never bid when the wind won't blow. This makes the storage useless—you never need to store power in the first place. Time-varying power prices can give value to storage, but that is outside the scope of this model.

It is then clear that $W_{\text{forecast}}(k) = kMp$ if there are k periods left, each one has the probability of generating power p, and the per-unit sale price of the power is M. Eq. (20) gave the value of a pure wind producer with no storage and no forecast skill. In that case, the wind project had a value of either $W(k) = kM[p - x(1-p)]$ if $x < p/(1-p)$, or $W(k) = 0$ if $x \geq p/(1-p)$.

Value of a Perfect Forecast to a Wind Producer with No Storage

Relative to the pure wind producer, then, the wind producer with a perfect one-period forecast can extract the value of

$$FV = W_{\text{forecast}}(k) - W(k) = kxM(1-p)[x < p/(1-p)] \quad (21)$$

in the low-penalty case where $x < p/(1-p)$ and extract a (higher) value of

$$FV = W_{\text{forecast}}(k) - W(k) = kMp[x \geq p/(1-p)]$$

in the high-penalty case of $x > p/(1-p)$.

(It should be underlined that the dividing line between high and low penalties is very different between the storage and no-storage cases.)

Value of a Perfect Forecast to a Wind Producer with Storage

Now let's look at the value of the perfect-forecast case relative to the value of the storage facility. In other words, what additional value does a perfect forecast bring to the operator of the coupled wind/storage project described earlier in this paper. There already is a strong intuition that this value will be less than in the no-storage case, since the storage allows penalties to be avoided much of the time.

This value is divided into three cases, as described next.

Small Penalty

$$x < \gamma p < p/(1-p){:}V(E,k) = kM[p - x(1-p)]$$

Here, $FV = kMx(1-p)$, the same as its value relative to the no-storage case. The associated bidding rule here is always to bid, whether empty or full. That means that the storage never will be filled, so no behavior changes relative to not having it. Note that if $x = 0$ (no penalties for nondelivery), the forecast has no value here, since simply always bidding will do the job just fine.

Medium Penalty

$$\gamma p \leq x < \gamma p/(1-p)$$

Here

$$V(E,k) = kpM - k^*Mx(1-p) - (k-k^*)M\gamma p(1-p),$$
$$k > k^* = kM[p - x(1-p)], \qquad k < k^*$$

where k^* is the largest integer satisfying

$$k^* < [\ln(x - p\gamma) - \ln x]/(\ln p)$$

This implies that

$$FV = k^*Mx(1-p) + (k-k^*)M\gamma p(1-p), \qquad k > k^*$$
$$FV = kMx(1-p), \qquad k < k^*$$

The rule here is to begin by bidding when full, not when empty, but eventually just always bidding.

Large Penalty

$$x \geq \gamma p/(1-p)$$

Here

$$V(E,k) = kpM[1 - \gamma(1-p)]$$

so

$$FV = kpM\gamma(1-p)$$

Note that this means if storage is free, there is no value to a perfect forecast. This makes sense because the associated bidding rule here, "Always bid when full, never bid when empty," will end up giving the ability to (1) never pay a penalty and (2) never waste any wind.

Of course, it is also of interest to determine the value of an energy storage facility to an operator with access to an imperfect wind forecast. The value of the storage in this case would be below its value given no forecast, but above its value given a perfect forecast. One way to think about valuing a storage given an imperfect forecast is as follows. This forecast would divide the operating hours of the facility into two groups. In the first, when wind is forecast, the conditional probability of wind would be higher than p; in the second, when calm is forecast, the conditional probability of wind would fall below p. Because of the strong serial independence assumption made in this paper, these two sets could be considered nearly independently of one another and each analyzed with with the results developed in this paper. For instance, if a forecast was very accurate, it might be that the conditional probability of wind given a wind forecast was high enough that it was optimal always to offer power, making a storage of no value, while the conditional probability of wind given a forecast of calm was so high that it as optimal never to offer power in the absence of storage. In this case, the storage value would come only from a reduced number of hours relative to a relatively low conditional wind probability. Working out all the details of this process is the topic of current work by the authors.

Discussion and Conclusions

Results analyzed in the previous section provide three primary insights on the use of storage with wind generation. First, given storage capabilities, the wind generator has incentive to behave in a manner that is helpful to the system; to bid into the market when there is likely to be wind and to drain storage to cover the shortfall if the wind doesn't manifest. This desirable behavior is achieved with a remarkably small penalty level. With the introduction of a time-varying probability of wind, akin to a diurnal weather pattern, the strategy holds, with the modification that the wind generator will bid into the market with an empty battery if the probability of wind is sufficiently high relative to the penalty. Second, in the absence of storage capability, the presence of a penalty would not achieve the desired effect. In such a case, the wind generator will simply pay the penalty if the resource is sufficient to make it economically viable and shut down if it is not good enough to overcome the burden of the penalty.

Finally, it is also shown that achieving better forecasting accuracy improves the wind resource and the value of the wind farm but also devalues the storage facility. There is no need for storage if the forecast is very accurate, except in the case of time-varying prices providing arbitrage opportunities, not discussed here. It is important to note that though there is much research interest and investment in improving battery efficiency, the findings here are relatively insensitive to the efficiency of the battery. Even a perfectly efficient battery will not change the behavior of the wind generator in response to the penalty structure.

In conclusion, with a relatively stylized model, we have gained significant insight into the optimal strategy under a range of conditions for wind, storage efficiency and shortfall penalties. This model can be generalized to model more realistic behaviors with numerical solutions. The importance of using the appropriate market tools, such as shortfall penalty, cannot be underestimated. This is a very efficient and important way to encourage wind generators to be constructive members of the power system, which will become increasingly more important as the penetration of wind increases in the future.

Appendix I. Proof of Theorem

Theorem 7.1. For the system of equations given below

$$\begin{aligned}
\boldsymbol{V(E,N,k)} &= \boldsymbol{pV(F,k}-1) + (1-\boldsymbol{p})\boldsymbol{V(E,k}-1)\\
\boldsymbol{V(E,B,k)} &= \boldsymbol{p}[\boldsymbol{M} + \boldsymbol{V(E,k}-1)] + (1-\boldsymbol{p})[-\boldsymbol{xM} + \boldsymbol{V(E,k}-1)]\\
\boldsymbol{V(E,k)} &= \mathbf{max}[\boldsymbol{V(E,N,k)},\boldsymbol{V(E,B,k)}]\\
\boldsymbol{V(F,N,k)} &= \boldsymbol{V(F,k}-1)\\
\boldsymbol{V(F,B,k)} &= \boldsymbol{p}[\boldsymbol{M} + \boldsymbol{V(F,k}-1)]\\
&\quad + (1-\boldsymbol{p})\,\mathbf{max}[(1-\boldsymbol{\gamma})\boldsymbol{M} + \boldsymbol{V(E,k}-1),\\
&\quad -\boldsymbol{xM} + \boldsymbol{V(F,k}-1)]\\
\boldsymbol{V(F,k)} &= \mathbf{max}[\boldsymbol{V(F,N,k)},\boldsymbol{V(F,B,k)}]
\end{aligned}$$

with initial conditions

$$\boldsymbol{V(E,}0) = 0 \qquad \boldsymbol{V(F,}0) = (1-\boldsymbol{\gamma})\boldsymbol{M}$$

the following result holds:

$$\boldsymbol{V(F,k)} - \boldsymbol{V(E,k)} = (1-\boldsymbol{\gamma})\boldsymbol{M} + \mathbf{min}[\boldsymbol{p\gamma M},(1-\boldsymbol{p^k})\boldsymbol{xM}],\quad \forall\ \boldsymbol{k} \geq 0$$

In order to prove this theorem, first show that the difference equation can be solved for $\boldsymbol{m(k)}$ defined here in closed form.

Lemma 7.1. Let $\boldsymbol{V(F,k)} - \boldsymbol{V(E,k)} = \boldsymbol{Mm(k)} + (1-\boldsymbol{\gamma})\boldsymbol{M}$. Then

$$\begin{aligned}
\boldsymbol{m(k)} &= \boldsymbol{m(k}-1) + \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}), \quad \boldsymbol{\gamma p} - \boldsymbol{pm(k}-1)]\\
&\quad - \mathbf{min}[\boldsymbol{p},\boldsymbol{x}(1-\boldsymbol{p}),(1-\boldsymbol{p})\boldsymbol{m(k}-1)]\\
\boldsymbol{m}(0) &= 0
\end{aligned} \tag{A1}$$

Proof. From the earlier result, now specialized for constant $\boldsymbol{p_k} = \boldsymbol{p}$, we have

$$\begin{aligned}
\boldsymbol{m(k)} &= \boldsymbol{m(k}-1) + \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),\boldsymbol{p\gamma} - \boldsymbol{pm(k}-1)]\\
&\quad - \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),(1-\boldsymbol{p})\boldsymbol{m(k}-1),\boldsymbol{p}]
\end{aligned}$$

With this in place, it is clear that if it can be shown that $\boldsymbol{m(k)} = \mathbf{min}[\boldsymbol{p\gamma},(1-\boldsymbol{p^k})\boldsymbol{x}]$ satisfies the equation of the lemma, then the proof of the theorem is complete. To show this, use induction.

Proof. First, show that it holds for; $\boldsymbol{k} = 1$; i.e.,

$$\begin{aligned} \boldsymbol{m}(1) &= \boldsymbol{m}(0) + \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),\boldsymbol{\gamma p}] \\ &\quad - \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),(1-\boldsymbol{p})\boldsymbol{m}(0),\boldsymbol{p}] \\ &= 0 + \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),\boldsymbol{\gamma p}] - \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),0,\boldsymbol{p}] \\ &= \mathbf{min}[\boldsymbol{x}(1-\boldsymbol{p}),\boldsymbol{\gamma p}] \end{aligned}$$

which is clearly satisfied by $\boldsymbol{m}(1) = \mathbf{min}[\boldsymbol{\gamma p},(1-\boldsymbol{p}^1)\boldsymbol{x}]$. Next, assume that it holds for $\boldsymbol{k} \geq 1$ and show that it holds for $\boldsymbol{k}+1$. We know that either $\boldsymbol{x}(1-\boldsymbol{p}^k) \leq \boldsymbol{\gamma p}$ or $\boldsymbol{x}(1-\boldsymbol{p}^k) > \boldsymbol{\gamma p}$. Let's look at each case separately.

1. Case 1:
   $x(1-p^k) \leq \gamma p$, so $m(k) = x(1-p^k)$
   First, take

$$\begin{aligned} m(k+1) &= m(k) + \min[x(1-p), p\gamma - pm(k)] \\ &\quad - \min[x(1-p), (1-p)m(k), p] \\ &= x(1-p^k) + \min[x(1-p), \gamma p - px(1-p^k)] \\ &\quad - \min[x(1-p), (1-p)x(1-p^k), p] \end{aligned}$$

Now, write

$$\begin{aligned} &\min[x(1-p), \gamma p - px(1-p^k)] \\ &\quad = -px(1-p^k) + \min[x(1-p) + px(1-p^k), \gamma p] \end{aligned}$$

and

$$\begin{aligned} &\min[x(1-p), (1-p)x(1-p^k), p] \\ &= (1-p)x(1-p^k) + \min[x(1-p) - (1-p)x(1-p^k), \\ &\quad 0, p - (1-p)x(1-p^k)] \end{aligned}$$

Therefore

$$\begin{aligned} m(k+1) &= x(1-p^k) - px(1-p^k) - (1-p)(1-p^k) \\ &\quad + \min[x(1-p) + px(1-p^k), \gamma p] \\ &\quad - \min[x(1-p) - (1-p)x(1-p^k), \\ &\quad 0, p - (1-p)x(1-p^k)] \\ &= \min[x(1-p^{k+1}), \gamma p] \\ &\quad - \min[x(1-p) - (1-p)x(1-p^k), \\ &\quad 0, p - (1-p)x(1-p^k)] \end{aligned}$$

If it can be shown that $\min[x(1-p) - (1-p)x(1-p^k), 0, p - (1-p)x(1-p^k)] = 0$, then it is done. Starting with the first term, simplify as follows:

$$\begin{aligned} x(1-p) - (1-p)x(1-p^k) &= x(1-p)[1-(1-p^k)] \\ &= x(1-p)p^k > 0 \end{aligned}$$

Moving on to the third term, note that because it has been assumed that $x(1-p^k) \leq \gamma p$, then

$$\begin{aligned} -x(1-p^k) &\geq -\gamma p - (1-p)x(1-p^k) \\ &\geq -(1-p)\gamma p \\ p - (1-p)x(1-p^k) &\geq p - (1-p)\gamma p \\ &= p[1-(1-p)\gamma] = p^2\gamma > 0 \end{aligned}$$

Therefore

$$\min[x(1-p) - (1-p)x(1-p^k), 0, p - (1-p)x(1-p^k)] = 0$$

and finally,

$$\begin{aligned} m(k+1) &= \min[x(1-p^{k+1}), \gamma p] - 0 \\ &= \min[x(1-p^{k+1}), \gamma p] \end{aligned}$$

2. Case 2:
   $x(1-p^k) > \gamma p$, so $m(k) = \gamma p$.
   Now, take

$$\begin{aligned} m(k+1) &= m(k) + \min[x(1-p), p\gamma - pm(k)] \\ &\quad - \min[x(1-p), (1-p)m(k), p] \\ &= \gamma p + \min[x(1-p), \gamma p - p^2\gamma] \\ &\quad - \min[x(1-p), (1-p)\gamma p, p] \end{aligned}$$

Because $p > (1-p)\gamma p$, it is clear that $\min[x(1-p), (1-p)\gamma p, p] \neq p$. Therefore,

$$\begin{aligned} m(k+1) &= \gamma p + \min[x(1-p), (1-p)\gamma p] \\ &\quad - \min[x(1-p), (1-p)\gamma p] \\ &= \gamma p \end{aligned}$$

The final question is, does $\min[x(1-p^{k+1}), \gamma p] = \gamma p$? By the assumption in this paper, $\min[x(1-p^k), \gamma p] = \gamma p$. But it also is clear that

$$\begin{aligned} p^{k+1} < p^k - p^{k+1} &> -p^k \\ 1 - p^{k+1} &> 1 - p^k \\ x(1-p^{k+1}) &> x(1-p^k) \end{aligned}$$

Therefore, $\min[x(1-p^{k+1}), \gamma p] = \gamma p$ and we are done.

Appendix II. Optimal Bidding Rules

Up to this point, this paper has proved that it is better to use the storage than to pay a nonnegative penalty, no matter how small: result 1. Now, this appendix proves result 2, about bidding when the facility is full. Use that result to simplify $V(F,k) = \max[V(F,B,k), V(F,N,k)]$ to $V(F,k) = V(F,B,k)$; in other words, when full, a decision to bid should always be made.

Corollary 8.1. $\boldsymbol{V}(\boldsymbol{F},\boldsymbol{B},\boldsymbol{k}) \geq \boldsymbol{V}(\boldsymbol{F},\boldsymbol{N},\boldsymbol{k}), \forall\ \boldsymbol{k}$.

Proof. Given the general model, as well as the previous corollary, write

$$
\begin{aligned}
V(F,B,K)-V(F,N,k)=&\,p[M+V(F,k-1)]\\
&+(1-p)[(1-\gamma)M+V(E,k-1)]\\
&-V(F,k-1)\\
=&\,pM+(1-p)\{(1-\gamma)M\\
&-[V(F,k-1)-V(E,k-1)]\}\\
=&\,pM-(1-p)\min[p\gamma M,(1-p^{k-1})xM]>0
\end{aligned}
$$

giving the desired result $V(F,B,k) > V(F,N,k)$.In other words, when full, it is always optimal to bid, regardless of x.The following corollary determines the bidding rules for an empty storage facility.

Corollary 8.2. When empty, it is better to bid if and only if $\gamma p > x(1-p^k)$.

Proof. Write

$$
\begin{aligned}
&V(E,B,k)-V(E,N,k)\\
&=p[M+V(E,k-1)]+(1-p)[-xM+V(E,k-1)]\\
&\quad -pV(F,k-1)-(1-p)V(E,k-1)\\
&=pM-p[V(F,k-1)-V(E,k-1)]-(1-p)xM\\
&=M[p-x(1-p)]-p[(1-\gamma)M+\min[p\gamma M,(1-p^{k-1})xM]\\
&=M[\gamma p-x(1-p)]-pM\min[p\gamma,(1-p^{k-1})x]\\
&=M[\gamma p-x(1-p)]+pM\max[-p\gamma,-x(1-p^{k-1})]\\
&=M\max[\gamma p-x(1-p)-\gamma p^2,\gamma p-x(1-p^k)]\\
&=M\max[(\gamma p-x)-p(\gamma p-x),\gamma p-x+xp^k)]
\end{aligned}
$$

If $x < \gamma p$ then $-p(\gamma p-x) < 0$. Therefore,

$$
\begin{aligned}
V(E,B,k)-V(E,N,k)&=M(\gamma p-x)+Mxp^k\\
&=M[\gamma p-x(1-p^k)]
\end{aligned}
$$

If $\gamma p < x$, then this can be rewritten as

$$
V(E,B,k)-V(E,N,k)=M\max[(1-p)(\gamma p-x),\gamma p-x+xp^k)]
$$

The first term is certainly negative because $x > \gamma p$, and the second term is positive if $\gamma p > x(1-p^k)$.

In both cases, if $\gamma p > x(1-p^k)$, then it is better to bid, as desired.

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