



# Using real option analysis to quantify ethanol policy impact on the firm's entry into and optimal operation of corn ethanol facilities



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## ABSTRACT

Ethanol crush spreads are used to model the value of a facility which produces ethanol from corn. A real option analysis is used to investigate the effects of model parameters on the related managerial decisions of (i) how to operate the facility through optimal switching from idled to operational status and (ii) the decision to enter into the project given its expected real option net present value. We present evidence of increased correlation between corn and ethanol prices, perhaps as a result of government policy which has induced more players to enter into the market. This paper investigates the subsequent negative effects on firms. Further, this paper illustrates the impact of an abrupt change in government policy, as what happened in January 2012, on a firm's decision to enter the business.

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## 1. Introduction

In recent decades, efforts to promote US energy independence from foreign oil (110th Congress, 2007; Senate, 2012) and initiatives to obtain more fuel from environmentally friendly sources have led to increased subsidies to the production of ethanol from corn. These projects have been subsidized via policies such as the volumetric ethanol excise tax credit provided to domestic ethanol biofuel blenders and the small ethanol producer tax credit. The amount of subsidy for blenders has changed from \$0.40/gal in 1978 (Energy Tax Act) to \$0.60/gal at its peak in 1984 (Tax Reform Act) (EIA, 2012). With the introduction of the 2008 Farm Bill, the subsidy was reduced to \$0.45/gal and many subsidies expired at the beginning of 2012. Before its December 2011 expiry, the small ethanol producer tax credit was \$0.10/gal. This credit applied to the first 15 million gal of annual production for a producer whose capacity did not exceed 60 million gal (DOE, 2013).

The corn ethanol process has been criticized on several grounds. Environmental critics claim that the process is energy negative in that more carbon-based energy is used to grow and convert the corn into ethanol than is produced through the process (Patzek et al., 2005; Pimentel, 2003). Public choice critics claim that the ethanol subsidies may be a result of seeking rents and lobbying (Spoel, 2009; Yandle, 1999) or that production must receive subsidies to become sufficiently economically attractive (a point discussed in this paper). Other critics claim that diverting corn from food or feed consumption has caused an increase in food prices and price variability (Elobeid et al., 2006; McPhail and Babcock, 2008, 2012) and that ethanol subsidies have other effects on social welfare (de Gorter and Just, 2010). A year following the lapse of these subsidies, about one quarter of Nebraska's ethanol plants were in idle status (NEB, 2013). It is possible that reduction in ethanol policy was a contributing factor.

In this paper, real option analysis is used to assess the optimal operating strategy for an ethanol production facility from management's perspective. In addition, the firm's decision of when to optimally enter the business of ethanol production is also analyzed. The model aims to realistically capture the flexibility inherent in the full life of the project through the ability to switch production on and off. There is a cost associated with switching production which means management faces a

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“wait and see” period before making a decision to change production. This resulting decision is non-trivial and must be modeled as a stochastic optimal control problem.

This real option method of modeling resource project management decisions was introduced by Brennan and Schwartz (1985) in a seminal paper which considered the problem of optimally starting and stopping production to maximize the profits of a natural resource project. The optimal entry and exit from investment projects was also considered by Dixit (1989) in another classical real option paper.

Our paper also investigates the effects of increased price correlation between ethanol and corn resulting from the diversion of corn from feedstock to fuel (Elobeid et al., 2006; McPhail and Babcock, 2008, 2012). Investigating correlation is expedient because it follows from straightforward economic arguments reducible to a single parameter. Earlier work in this area has focused on changes in correlation over time. Kirby and Davison (2010) suggest it may have increased; Schmit et al. (2011) present evidence that it may have decreased. In either scenario, investigating its impact on pricing and operating decisions is important. Further, the effects of policy subsidy on project value are also investigated.

This paper represents a direct extension of the analysis in Kirby and Davison (2010) and uses similar methods to those presented in Schmit et al. (2011). Kirby and Davison (2010) use a bootstrap Monte Carlo analysis to estimate the value of an ethanol production facility modeled as a strip of exchange options. Our paper expands the analysis in Kirby and Davison (2010) to capture the rational operator's optimal strategy which hinges on the “wait and see” phenomenon. Schmit et al. (2011) investigate the effects of ethanol policy on prices and the firm's decision to enter into and divest itself from the business on an infinite time horizon. This paper adds to their analysis by (1) using a finite time horizon for entry into the project in keeping with for instance a private equity firm's finite investment horizon requirements; and (2) investigating its subsequent optimal operation once initiated. The effect of ethanol policies on prices is investigated using increases in a simple ethanol–corn correlation metric designed to capture the increased linkage between the two markets.

Section 2 assembles a framework including price models, parameters for management flexibility and rules for optimal operation. Section 3 illustrates concepts and heuristic results from similar closed-form models while Section 4 contains the numerical results from the full analysis. Finally, Section 5 presents insights and conclusions from the investigation.

## 2. Assembling the model

Firms have the flexibility to begin or defer projects given current economic and price environments, a flexibility not captured by net present value (NPV) or discounted cash flow (DCF) analyses, as described in Dixit and Pindyck (1994).

After entering into an ethanol project, management has the ability to pause and resume production given price conditions and their profits. This enables management to capitalize on the upside profits while mitigating the downside losses. Again a simple NPV DCF analysis would fail to capture the true value of flexibility given uncertain (stochastic) future prices.

The goal of this paper is to examine how ethanol policy affects producers' business entry and subsequent facility operation decisions given price conditions, subsidy expectations, and the remaining facility life.

To develop a model, the following inputs are required:

1. Equations representing the plant economics including capitalized costs to construct the facility, costs to pause and resume production, and instantaneous profit as a function of ethanol and corn prices; and
2. A stochastic model for corn and ethanol prices including econometric analysis of the relevant parameters.

Throughout this paper, all currency units are United States dollars (USD); all liquid volume units are gallons (1 US gal = 3.785 L); all solid volume units are bushels (1 US bushel = 0.0352 m<sup>3</sup>); all weight units are in tons (1 short t = 2000 lb = 907.185 kg); and all interest rates are percent per year and appropriate to USD deposits.

### 2.1. A model for the plant

The plant produces ethanol (priced in USD/gal) from corn (priced in USD/bushel).

#### 2.1.1. Reaction models and instantaneous running profits

The running profit from the corn–ethanol crush spread is developed on a per bushel per year \$/bushel-year. Our analysis uses the standard reaction from Bothast and Schlicher (2005) for the popular dry grind process of producing ethanol

$$\text{corn} \rightarrow \text{ethanol} + \text{byproducts}, \quad (1)$$

which implies the profit function

$$f(L_t, C_t) = \kappa(L_t - K) + \omega A_t - C_t. \quad (2)$$

The net running cost  $K$  may further be decomposed into the fixed running cost  $p$  less any government volumetric subsidy  $s$ ,

$$K = p - s. \quad (3)$$

$L_t$  is the price of ethanol per gallon and  $A_t$  is the price of byproduct distillers dried grains in dollars per ton. The process produces 17 lbs of distillers dried grains per bushel of corn and consequently  $\omega = 17/2000$ .<sup>1</sup> The conversion factor,  $K = 2.8$ , represents how many gallons of ethanol are produced per bushel of corn; taken from Bothast and Schlicher (2005) and is consistent with the CME Group's references on ethanol crush spreads (CME, 2010). A subsidy of \$0.10/gal was used along with a fixed running cost of \$0.68/gal for facilities with nameplate capacities of 40,000,000 gal/year (Schmit et al., 2009).

The analysis is simplified by considering two stochastic factors, ethanol and corn, independently; while accounting for each additional factor with affine terms. This yields a simple instantaneous running profit function

$$f_1(L_t, C_t) = \kappa(L_t - K_1) - C_t \quad (4)$$

on a per bushel consumed per year basis. Average distillers dried grains,  $\bar{A}_t$ , is one constituent of the parameter  $K_1$

$$K_1 = p - \frac{\omega}{\kappa} \bar{A}_t - s. \quad (5)$$

While production is idle, Schmit et al. (2009) estimated that fixed running costs are roughly 1% of capitalized cost per gallon of capacity,  $B$ , or roughly 20% of fixed running cost while in production. Our analysis takes the average of these two fixed running cost estimates. While production is halted there is no subsidy since no ethanol is being produced. The profit function while off is

$$f_0(L_t, C_t) = -\kappa K_0 \quad (6)$$

where

$$K_0 = \left( \frac{0.20p + 0.01B}{2} \right) \quad (7)$$

is the midpoint of the two possible estimates of  $K_0$ .

<sup>1</sup> There are 2000 lbs in a ton and distillers dried grain prices are quoted in USD/t.

2.1.2. Switching and capitalized construction costs

For a medium-sized facility (40,000,000 gal/year) Schmit et al. (2011) estimated a capitalized cost of \$1.40/gal is required to construct a turn-key facility from a green field. The medium-sized facility is taken as the representative model which also qualifies for the small ethanol producer subsidy being less than 60,000,000 gal in capacity. Costs to resume production from an idle state are estimated by Schmit et al. (2009) to be 10% of capitalized cost per gallon of capacity; costs to pause production from an active state are estimated to be 5% of capitalized cost; finally the liquidation value at the end of facility life is estimated to be 10% of capitalized cost.

2.2. Models of the prices

Ethanol and corn are modeled as stochastic geometric Brownian motion (GBM) processes in this analysis. Despite some well-known drawbacks, GBM is very popular in mathematical finance and financial economics due to its simplicity and robustness for modeling financial time series. The historical price series from Dec/02–Jan/11 is shown in Fig. 1.

A GBM random process  $X_t$  follows the stochastic differential equation (SDE):

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \tag{8}$$

where  $\mu$  is its drift (average rate of continuously compounded growth) and  $\sigma$  is its volatility. The differential increment of Brownian motion  $dW_t$  corresponding to the interval between  $t$  and  $t + dt$  is drawn from the normal random variable with zero mean and variance  $dt$ , independent of other such draws on non-overlapping time intervals.

This model is reasonable since statistical tests on the time series in Kirby and Davison (2010) rejected mean-reversion and seasonality although it was found that the data exhibit serial autocorrelation. The effects of autocorrelation in the drift of the lagged process were found to be statistically zero in Schmit et al. (2011) and hence serial correlation is ignored in our paper’s analysis; the time series were also subjected to augmented Dickey–Fuller tests which found weak evidence against the presence of unit roots and hence the time series can be treated as stationary. This also allows the use of well-developed theory of Markov processes and Ito calculus in the analysis that follows.

The logarithm of a GBM process  $\ln X_t$  follows an even simpler constant volatility arithmetic Brownian motion (ABM) process

$$d \ln X_t = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t. \tag{9}$$

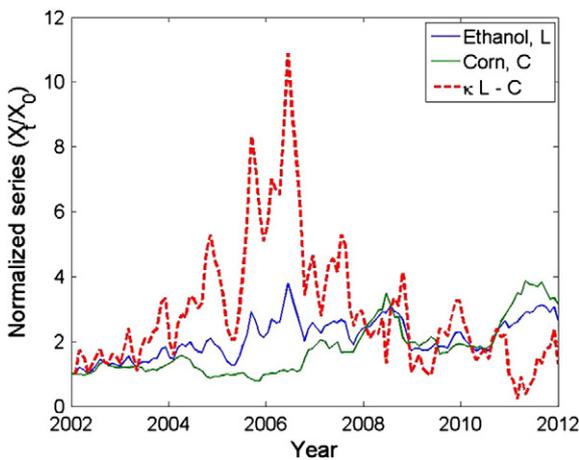


Fig. 1. Historical ethanol–corn price series from Jan/02 to Dec/11.

The econometric parameters are estimated by ordinary least-squares regression. The differenced ABM series  $\Delta \ln X_t = \ln \frac{X_t}{X_{t-1}}$  has representation

$$\Delta \ln X_t = \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \sqrt{\Delta t} \xi \tag{10}$$

where  $\Delta t = t_i - t_{i-1}$  and  $\xi \sim N(0,1)$ . Thus the parameters may now be estimated via

$$\Delta \ln X_t = \beta_0 + \epsilon \xi, \tag{11}$$

where the constant term is the drift of the series and the volatility is read directly from the root mean squared error of the innovation  $\epsilon \xi$

$$\beta_0 = \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t, \tag{12}$$

$$RMSE = \sqrt{\epsilon} = \sqrt{\Delta t} \sigma. \tag{13}$$

Estimates of the correlation,  $\rho$ , between two time series are obtained via the sample correlation of the residuals.

Prices for the no. 2 Omaha, Nebraska yellow corn used to underpin the standard CME contract were obtained from the US Department of Agriculture feed grains database (USDA, 2012). Average rack price freight on board ethanol prices was obtained from the Nebraska Energy Office (NEO, 2012). Nebraska data was selected to reflect the size of Nebraska in US corn and ethanol markets and to be reflective of national prices (NCB, 2012). To be consistent with Kirby and Davison, 10 years of monthly historical price data was used spanning the period between Jan/02 and Dec/11.

For simplicity, the relatively small inflation adjustment for prices over this 10 year period was ignored. Note that inflation enters into the price dynamics via the drift, the specification of which does not affect the estimates for volatility and correlation.

Ethanol and corn are modeled by correlated GBMs with SDEs

$$dL_t = \mu L_t dt + \sigma L_t dW_{1t}, \tag{14}$$

$$dC_t = a C_t dt + b C_t dW_{2t}, \tag{15}$$

$$\text{Corr}[W_{1t}, W_{2t}] = \rho. \tag{16}$$

The drifts of both ethanol and corn were found to be statistically zero at the 95% confidence interval. The annualized results are summarized in Table 1.

The estimate for the average distillers dried grains price  $\bar{A}_t$  was estimated by regressing the time series against a constant. The result was  $\bar{A}_t = 115.6$  with a standard error of the estimator (*s.e.*) of 3.6. At the 95% student-*t* percentile with 119 degrees of freedom,  $t_{0.975,119} = 1.9801$ , the confidence interval is  $\bar{A}_t \in [108.4, 122.8]$ .

2.3. The real option model

Now a model is developed for the optimal operating strategy and expected earnings of the plant. All earnings are discounted using an

Table 1  
Parameter estimation results.

Parameter estimate	Value	t-test
$\hat{\mu}$	0	$P\left(\frac{\hat{\mu}-\mu}{s.e.} >  t  \mid \mu = 0\right) = 0.409$
$\hat{\sigma}$	0.156	–
$\hat{a}$	0	$P\left(\frac{\hat{a}-a}{s.e.} >  t  \mid a = 0\right) = 0.202$
$\hat{b}$	0.123	–
$\hat{\rho}$	0.105	–

annualized interest rate of  $r = 8\%$  which aims to capture the credit risk associated with ethanol projects (Schmit et al., 2009).

The plant has two operating modes or states: 1, denoting “on” or in production, and 0, denoting “off” or production temporarily suspended. The instantaneous running profit while on is given by  $f_1$ ; while off by  $f_0$ . The cost of switching production back on after being temporarily suspended is  $D_{01}$  and the cost of switching production off from an active state is  $D_{10}$ .

The capitalized cost of construction of the facility is given by  $B$  and its liquidation value at the end of its normal useful life is  $Q$ . All parameter and function values are listed in Table 2.

The total expected earnings over the life of the project is given by the value function  $V_i$  where  $i = \{1,0\}$

$$V_i(l, c, t) = \sup_{\tau, u} E \left[ \int_t^T e^{-r(s-t)} f_i(L_s, C_s) ds + \sum_{k=1}^n e^{-r(\tau_k-t)} D_{u_{k-1}, u_k} \middle| (L_t, C_t, u_0) = (l, c, i) \right] \tag{17}$$

The pair  $(\tau, u)$  is the control that the manager has over the facility in his ability to toggle production on and off. It consists of a set of switching times  $\tau_k$  and states to be switched into  $u_k$  with  $L_t = u_k, t \in [\tau_k, \tau_{k+1})$ . Thus  $\tau_k$  is an increasing set of switching times with  $\tau_k \in [t, T]$  and  $\tau_k < \tau_{k+1}$ .

From the dynamic programming principle, it is known that

$$V_i(l, c, t) = \sup_{\tau} E \left[ \int_t^{\tau} e^{-r(s-t)} f_i(L_s, C_s) ds + e^{-r(\tau-t)} \{V_j(L_{\tau}, C_{\tau}, \tau) - D_{ij}\} \middle| (L_t, C_t) = (l, c) \right] \tag{18}$$

where  $\tau$  is the optimal time switch production on from off ( $0 \rightarrow 1$ ) or off from on ( $0 \rightarrow 1$ ).

This problem can be reduced to a question of finding the optimal price boundaries for ethanol and corn ( $L_t, C_t$ ) at which to switch production. Now the problem is to solve for the sets of prices at which the operator should:

- continue production if production is currently on,  $H_1$ ;
- pause production if the state is currently on,  $S_{10}$ ;
- keep production halted if the state is currently idle,  $H_0$ ; and
- resume production if the state is currently idle,  $S_{01}$ .

Thus given the production is in state  $i$  at time  $t$  only one of two decisions is possible. (1) If it is optimal to keep production in its current state, then by Ito’s lemma the value function evolves by the partial differential equation (PDE) on  $H_i$

$$\frac{\partial V_i}{\partial t} + L[V_i] + f_i(l, c, t) - rV_i = 0, \tag{19}$$

where  $L$  is the generator of the joint processes  $(L_t, C_t)$

$$L = \mu l \frac{\partial}{\partial l} + ac \frac{\partial}{\partial c} + \frac{1}{2} \sigma^2 l^2 \frac{\partial^2}{\partial l^2} + \rho \sigma b l c \frac{\partial^2}{\partial l \partial c} + \frac{1}{2} b^2 c^2 \frac{\partial^2}{\partial c^2}. \tag{20}$$

Similarly (2) if it is optimal to switch (if the value of the  $i$ -state were to fall below the  $j$ -state less switching costs  $V_i(l, c, t) \leq V_j(l, c, t) - D_{ij}$ ) then immediately

$$V_i(l, c, t) = V_j(l, c, t) - D_{ij} \tag{21}$$

on  $S_{ij}$  and the operator switches to receive the profits in state  $j, f_j$ .

This leads to the set of free boundary PDEs for the optimal switching problem. The free boundary  $\partial H_i$  gives the optimal set of prices at which to toggle production. In order to solve the PDE, the moving free boundary must be determined; it is not known a priori. Along the free boundary, there is continuity of the value functions and its first spatial derivatives, the so-called “high contact” principle (Brekke and Øksendal, 1994). By writing the free boundary problem in complementary form below (noting that either the PDE holds or the constraint is saturated), it is no longer necessary to track the free boundary as the equation is extended to the whole space.

$$\max \left[ \frac{\partial V_1}{\partial t} + L[V_1] + f_1(l, c, t) - rV_1, (V_0 - D_{10}) - V_1 \right] = 0, \tag{22}$$

$$\max \left[ \frac{\partial V_0}{\partial t} + L[V_0] + f_0(l, c, t) - rV_0, (V_1 - D_{01}) - V_0 \right] = 0 \tag{23}$$

with final conditions  $V_1(l, c, T) = V_0(l, c, T) = Q$ .

These equations may be solved numerically using methods similar to those described in Wilmott et al. (1994). The PDE component is solved using finite differences using a standard elliptic solver for the spatial components along with a time stepping discretization. The complementarity condition for the optimal switching is enforced using an iterative fixed point method. Conceptually, the technique is similar to projected successive over-relaxation (Cryer, 1971) and can be accelerated with multigrid or Krylov methods. Each system  $V_1, V_2$  is iterated simultaneously until convergence. For additional information on optimal switching problems and stochastic calculus, see Pham (2009), Bensoussan and Lions (1984), Øksendal (2007), and Brekke and Øksendal (1994).

Suppose the firm has a lease over a finite time horizon on the green field site on which they plan to build the production facility. If prices are particularly unfavorable, it would be naive to immediately enter into the project. A rational investor that seeks to maximize his expected earnings  $P$  should wait at least until the expected earnings of the optimally managed facility exceed the capital cost of investment. This is analogous to an American call option on the facility struck at  $B$  with

**Table 2**  
Real option model parameters.

Variable	Description	Value	Source
$B$	Capitalized cost of construction	\$1.40/gal	Schmit et al. (2009, 2011)
$Q$	Liquidation value at end of life	$0.1B = \$0.14/\text{gal}$	Schmit et al. (2009)
$D_{01}$	Cost to switch production on	$0.1B = \$0.14/\text{gal}$	Schmit et al. (2009)
$D_{10}$	Cost to switch production off	$0.05B = \$0.07/\text{gal}$	Schmit et al. (2009)
$r$	Discount rate	8% per annum	Schmit et al. (2011)
$p$	Fixed running cost	\$0.68/gal	Schmit et al. (2009)
$s$	Subsidy	\$0.10/gal	EIA (2012)
$\bar{A}_t$	Average price of distiller’s dried grains per ton	\$115.58/t	USDA (2012)
$\kappa$	Gallons of ethanol produced per bushel of corn	2.8 gal/bushel	Bothast and Schlicher (2005)
$\omega$	Tons of distillers dried grains produced per bushel of corn	$\frac{17}{2000}$ t/bushel	Bothast and Schlicher (2005)
$K_1$	Net running costs while in production	0.23/gal	Schmit et al. (2009)
$K_0$	Net running costs while idle	0.07/gal	Schmit et al. (2009)
$f_1(L_t, C_t)$	Running profits while in production	$\kappa(L_t - K_1) - C_t = 2.8(L_t + 0.12) - C_t/\text{bushel-year}$	
$f_0(L_t, C_t)$	Running profits (losses) while production is idle	$-K_0 = -0.07/\text{bushel-year}$	

payoff  $\max\{V_1(l, c, t), V_0(l, c, t) - B\}$  over the remaining horizon  $T - t$ . The free boundary problem for this option (following a similar dynamic programming optimal stopping argument) is

$$\max \left[ \frac{\partial P}{\partial t} + L[P] - rP, \max\{V_1(l, c, t), V_0(l, c, t) - B\} - P \right] = 0 \quad (24)$$

with final condition  $P(l, c, T) = \max\{0, \max\{V_1(l, c, T), V_0(l, c, T) - B\}\}$ . Again, this is reduced to finding a set on which it is optimal to wait,  $H$ , and set at which it is optimal to enter into the investment,  $S$ . The free boundary between these two sets is the set of prices at which it is optimal to make the decision. See Wilmott (2006) for additional details.

As the green field project is quite expensive to initiate relative to its salvage value upon abandonment, the option to abandon adds little value and for financially reasonable parameters does not materially alter the decision to enter the investment. A thorough argument is presented in Appendix A.

### 3. Lessons from exchange options

In this section, two simplifications of the above model are presented to predict the effects of increased correlation on the complete model.

#### 3.1. A running Margrabe exchange option

Assume that switching costs and fixed running costs are both zero. This makes it possible to find an analytic solution for the expected earnings of the facility. If switching costs are zero, the problem reduces to the simple PDE

$$\frac{\partial V}{\partial t} + L[V] + (\kappa l - c)^+ - rV = 0, \quad (25)$$

where  $V_1 = V_0 = V$  and  $X^+ = \max(X, 0)$ . This is the running payoff analogue of the classical Margrabe European exchange option (Margrabe, 1978).

The solution to this problem follows from the Feynman–Kac representation theorem

$$V(l, c, t) = E \left[ \int_t^T e^{-r(s-t)} (\kappa L_s - C_s)^+ ds \mid (L_t, C_t) = (l, c) \right]. \quad (26)$$

After some reflection, it is apparent that Eq. (26) is similar to a running Margrabe exchange option or a Black–Scholes call on ethanol struck at the corn price. Following the Black–Scholes analogy, Eq. (26) is reduced to

$$V(l, c, t) = \int_t^T e^{-r(s-t)} \left[ \kappa l e^{\mu(s-t)} \Phi(d_1) - c e^{a(s-t)} \Phi(d_2) \right] ds \quad (27)$$

where

$$v^2 = \sigma^2 - 2\rho\sigma b + b^2, \quad (28)$$

$$d_1 = \frac{\ln\left(\frac{\kappa l e^{\mu(s-t)}}{c e^{a(s-t)}}\right) + \frac{1}{2} v \sqrt{s-t}}{v \sqrt{s-t}}, \quad (29)$$

$$d_2 = \frac{\ln\left(\frac{\kappa l e^{\mu(s-t)}}{c e^{a(s-t)}}\right) - \frac{1}{2} v \sqrt{s-t}}{v \sqrt{s-t}}. \quad (30)$$

From Eqs. (27) to (30) above, it is apparent that  $v$  is decreasing in  $\rho$ . Since this is akin to a Black–Scholes option, its value is accordingly decreasing in  $\rho$ . Similarly it is approximately semilinear in  $c$ , the operating cost, deep into or out of the money. This is illustrated in Fig. 2 for a generic-parameter option in the risk neutral measure (where  $\kappa = 1$ ,  $L_t = 1$ ). Therefore, as a rough approximation,  $V$  can be considered almost semilinear and decreasing in  $K$ .

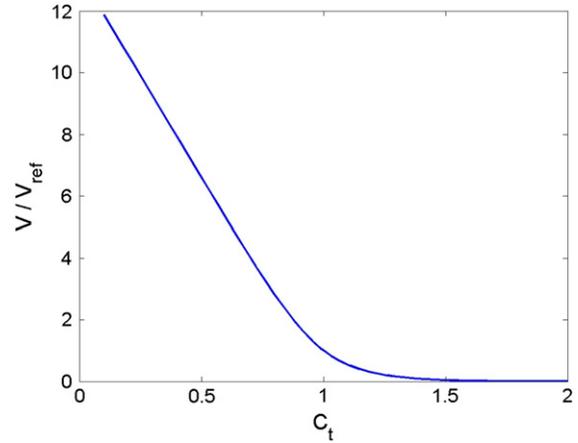


Fig. 2.  $V/V_{ref}$  as a function of  $C_t$  is approximately semilinear. All parameters are as in Table 2 except  $K = 0, D_{01} = D_{10} = 0$ , and  $\kappa = 1$ .

The value of the facility is also strongly linked to its achievable yield of ethanol per bushel of corn. As before, the value is semilinear in  $\kappa$ .

Fig. 3 shows the percent decrease in  $V$  at the money as a function of increasing  $\rho$  normalized by the reference value  $V_{ref} = V(\rho = 0)$ .

#### 3.2. An infinite horizon model

Using a clever dimensional reduction to obtain coupled differential equations, Schmit et al. (2011) were able to solve an infinite time horizon problem in closed form. Changing notation to that used in the current paper, their solution can be represented as the system of nonlinear equations

$$v_0(z) = Az^{\lambda_-}, \quad (31)$$

$$v_1(z) = Bz^{\lambda_+} + \frac{z}{r-\mu} - \frac{1}{r-a}, \quad (32)$$

$$\lambda_{\pm} = \left( \frac{1}{2} - \frac{\mu-a}{v^2} \right) \pm \sqrt{\left( \frac{\mu-a}{v^2} - \frac{1}{2} \right)^2 + \frac{2(r-a)}{v^2}}, \quad (33)$$

where  $V(l, c) = cv(\frac{l}{c}) = cv(z)$ .

The remaining four unknowns  $A$  and  $B$ , and  $z_{01}$  and  $z_{10}$ —which represent the  $z$  at which production should be switched on or off respectively—derive from continuity of the value functions and the smooth-pasting optimality condition (i.e. 1st derivatives) at the

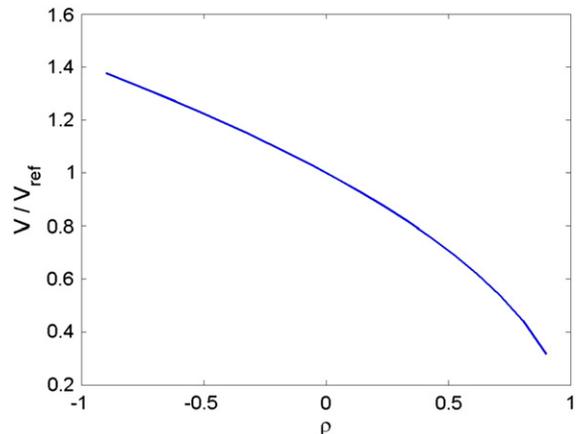


Fig. 3.  $V/V_{ref}$  as a function of  $\rho$  is decreasing. Parameters are as in Table 2 except  $K = 0, D_{01} = D_{10} = 0$ .

switching boundary which constitutes a system of four nonlinear equations in four unknowns.

The value function for the parameters calculated is shown in Fig. 4. The switching boundaries  $z_{01}, z_{10}$  as a function of  $\rho$  are shown in Fig. 5.

As can be seen from the figures, the effect of increasing  $\rho$  tightens the “wait-and-see” gap resulting in shorter periods of operation before making the decision to switch. In particular, the plant manager is less optimistic about prices rebounding in making the decision to switch production off. This is accompanied by decreased value and potentially riskier cash flows since production is started and stopped more often.

A technical term describing this gap phenomenon is hysteresis. It represents a “sticky” region where it is not definitively optimal to be in either state (on or off) but rather to remain operating as is. Once prices reach the switching boundaries  $S_{01}$  and  $S_{10}$ , it is definitively optimal to be in either the on or off state respectively and switching occurs as required. Stated precisely, the hysteresis zone is given by the set  $H_0 \cap H_1$  which is also equivalent to  $H_1 \setminus S_{10} = H_0 \setminus S_{01}$ .

**4. Numerical results**

The analysis begins with a retrospective look at the profits that would have been realized by the model facility given historical prices from Jan/02 to Dec/11. As a baseline, 10 year model values at the Jan/02 price of  $L_t = \$0.94/\text{gal}$  for ethanol and  $C_t = \$1.90/\text{bushel}$  for corn are listed in Table 3 ignoring the value of liquidating the plant at the end of its life (i.e.  $Q = 0$ ). As before,  $V_i(l, c, t)$  refers to the expected value of income generated by the facility from time  $t$ , given production begins in state  $i$ , with  $L_t = l$  and  $C_t = c$ .

The actual profits given the past 10 year time series from Jan/02 to Dec/11 realized from optimal operation are also recorded in Table 3, noted as  $V_i|_{\text{realized time series}}$ . The higher than expected realized profits do not reflect negatively on the model’s validity but rather represent one of many possible realized outcomes from the stochastic model.

The retrospective plant operating status from Jan/02 to Dec/11 as determined by following the optimal operating scheme indicates that the facility should always be in production regardless of price conditions (see Fig. 1). The results suggest that the ethanol subsidy policy may be higher than necessary to ensure NPV positivity and may in fact be reduced with minimal effects on producers.

**4.1. Baseline value**

The baseline valuation results are shown in Fig. 6 which include the liquidation value at the end of facility life on a per bushel basis.

The baseline switching ( $S_{01}, S_{10}$ ) and continuation sets ( $H_0, H_1$ ) are shown in Fig. 7.

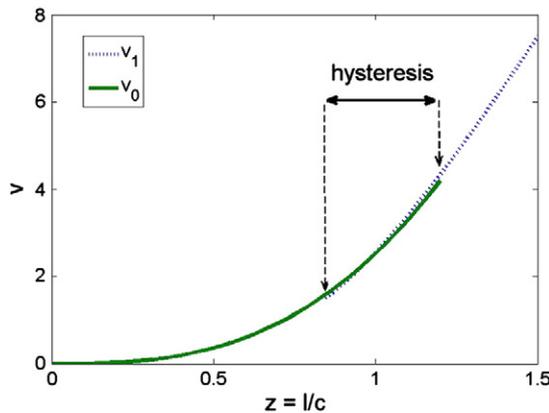


Fig. 4. Baseline results for  $v_0$  and  $v_1$ . Note the value is increasing in  $l/c$  and the presence of a hysteresis zone in the value functions. Parameters are as in Table 2 except  $T = \infty$ .

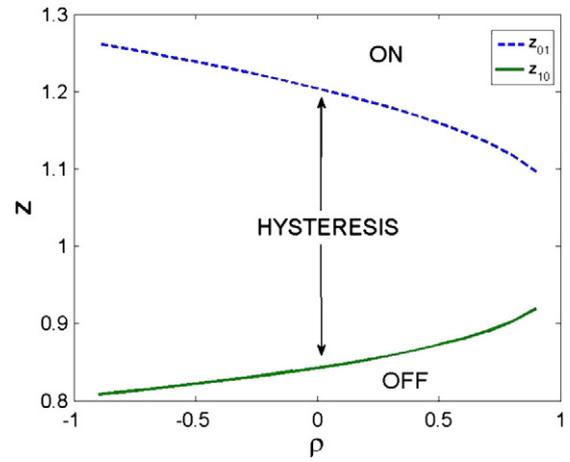


Fig. 5. Switching boundaries  $z_{01}$  and  $z_{10}$  as a function of  $\rho$  where  $z = l/c$ . Note that as  $\rho$  increases, the boundaries at which production is started  $z_{01}$  and stopped  $z_{10}$  converge indicating reduced “optimism” in prices rebounding. Parameters are as in Table 2 except  $T = \infty$ .

**4.2. Effects of increased correlation on value**

As expected from the Margrabe option results presented in Section 3, increasing correlation significantly reduces option value. There is evidence for increased correlation in recent years. Fig. 8 shows the rolling correlation over the previous 3 year period on the price series data from Jan/02 to Dec/11 calculated using the experimental correlation of the log monthly returns.

Fig. 9 shows the percent loss in income value as the correlation  $\rho$  increases, shown for 10 years of income without liquidation given the Jan/02 average monthly prices of  $(l, c) = (0.94, 1.90)$  with  $V_{i,ref} = V_i(p = 0)$ .

Pushing  $\rho$  away from zero correlation results in changes in  $\pm 50\%$  of income option value. The concavity of the graph indicates that the option is very sensitive to  $\rho$ .

Fig. 10 shows  $\frac{\partial V_i}{\partial \rho}$  evaluated at the estimated value of  $\rho = 0.105$  along with the switching boundaries overlaid on the plot. (The result for  $\frac{\partial V_0}{\partial \rho}$  is very similar.)

The effects of increasing  $\rho$  are strongest near the switching regions (i.e. in the hysteresis zone). The most significant losses in the hysteresis zone are near 0–0.50. Thus in these price regions for a 10% increase in  $\rho$ , there is a loss of nearly \$0.05 of value following the Taylor approximation,  $V(\rho_0 + \Delta\rho) = V(\rho_0) + \frac{\partial V}{\partial \rho} \Delta\rho + O(\Delta\rho^2)$ . Outside the hysteresis zone, all partials become equal since  $V_i = V_j - D_{ij}$  only differs by an additive constant. The hysteresis zone is the result of uncertainty as to which decision or operating status is optimal. Intuitively, the option value would be most sensitive to changes in variance (via correlation) in this uncertainty or hysteresis zone which is observed in Fig. 10.

The loss in value associated with increasing  $\rho$  becomes more persistent and pronounced as  $T - t$  increases. This is a financially intuitive result since there is more time for the losses related to  $\rho$  to accrue. But more importantly, the longer increased correlation persists the more damaging the effect becomes. To illustrate, Fig. 11 shows the relevant

**Table 3**

Expected income value as Jan/02 and retrospective historical realized income during the period Jan/02 through Dec/11.

Baseline result	Description	Value
$V_1(0.94, 1.90, 0)$	Model income value	\$2.10/bushel-year
$V_0(0.94, 1.90, 0)$	Model income value	\$1.77/bushel-year
$P(0.94, 1.90, 0)$	Model income value	\$0.84/bushel-year
$V_1 _{\text{realized time series}}$	Retrospective realized income	\$10.73/bushel-year
$V_0 _{\text{realized time series}}$	Retrospective realized income	\$10.27/bushel-year

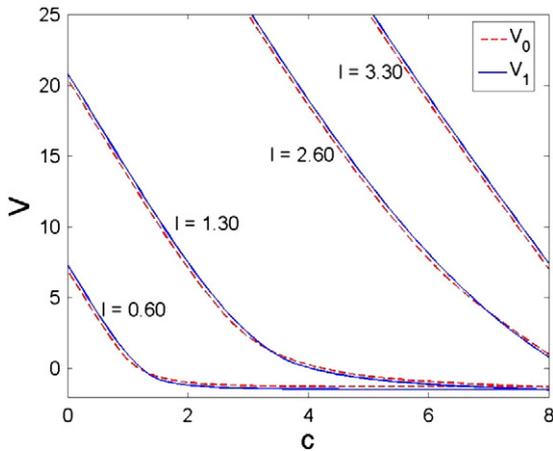


Fig. 6. Baseline income valuation results for  $V_1$  (solid) and  $V_0$  (dashed) for given levels of  $I$ . Note that  $I$  and  $c$  are the initial ethanol and corn prices given the 10 year period respectively. The  $y$ -axis is the income value. Parameters are as in Table 2.

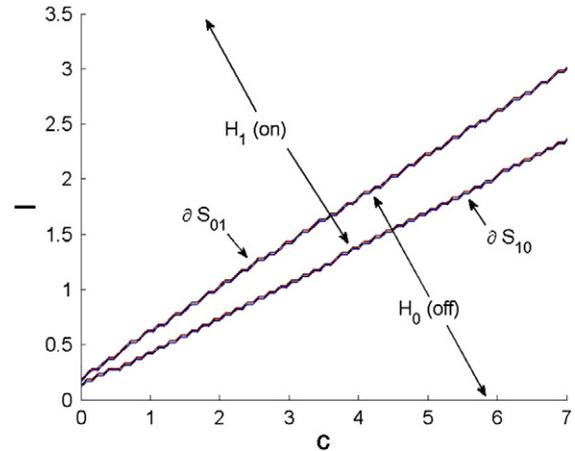


Fig. 7. Baseline switching,  $S_{ij}$ , and continuation sets,  $C_{ij}$ . Parameters are as in Table 2.

option “Greek,”  $\frac{\partial^2 V_1}{\partial t \partial \rho}$ , taken at Jan/02 price levels (0.94, 1.90) (note that similar results hold for  $V_0$ ).

4.3. Effects of subsidy policy on value

The loss in value from removing the subsidy is shown in Fig. 12. As the subsidy  $s$  is lowered, the facility is expected to lose value at a near linear rate when deep in the money and at a lower rate as the value moves further out of the money. This behavior is illustrated in Fig. 12 which plots  $V_i(s)$  at  $(l, c) = (0.94, 1.90)$ . It can also be seen from the plot that, across all  $s \in [0,10]$   $\text{¢/gal}$ , the point  $(l, c)$  remains in the hysteresis zone. The distance between the two values remains less than the switching cost  $V_1(l, c) - V_0(l, c) < D_{01}$ , a characteristic feature of the hysteresis zone.

4.4. NPV positivity and the value of waiting to invest

Since both increasing  $\rho$  and decreasing the subsidy have the effect of reducing the value of the income option, it is natural to expect that the value of waiting to invest,  $P$ , is also reduced. If the value is reduced, it is expected that the optimal price levels to begin the project,  $\partial S$ , should be closer to the NPV positive region.<sup>2</sup> This reflects the lowered optimism of entry into the investment. This is illustrated with exaggeration in Figs. 13, 14 and 15.

As is apparent from Fig. 15, the subsidy policy results in some otherwise economically unattractive projects being initiated. The net effect of this is to reduce the productive activity of firms contemplating entry into the investment project.

4.5. Retrospective analysis without subsidy

The investigation closes with a retrospective look at the performance of the optimal operating schedule without a subsidy; this gives an indication of the kind of performance one might expect in the future given many ethanol subsidies have been discontinued. Due to high realized ethanol prices, even a non-subsidized facility has a productive run and is nearly always in operation.

Contrast that operating result with Fig. 16 which indicates when the facility is operating at a profit, 1, or at a loss, 0. That is, Fig. 16 is a graph of  $1_{f_1(l,c) > 0}$  with and without subsidy. In the absence of a subsidy, it is still optimal to always remain in operation given the historical time series; despite the fact that on several occasions the profits become negative.

<sup>2</sup> The NPV positive region is given by the set at  $t = 0((l, c) : (\max[V_1, V_0] - B) > 0)$  over 10 years including liquidation proceeds.

The presence of switching costs acts like a low pass filter on the zero-cost switching signal and accordingly switching occurs less frequently.

It may appear that the facility is profitable even in the absence of subsidy, but part of the story is missing. Fig. 17 indicates when it is optimal to enter into the investment over the 10 year horizon. As can be seen, for most of the time it is in fact not optimal to initiate the project even though the retrospective operating status advises to be in production. This means that while the operator of an existing facility would produce from it, the resulting profits would not be so large as to entice the development of a new facility. It is in fact optimal to wait nearly 2 years before initiating the project even in spite of low corn prices and the otherwise continuous production signal. In addition, it is apparent that the presence of the subsidy does not greatly influence the historical decision to enter into the project.

The assumption that the facility is able to easily market and sell its distillers dried grains may not always hold. An investigation of the historical operating status given  $\bar{A}_t = 0$  is shown in Fig. 18. The upper indicator assumes the state is initially on,  $V_1, i = 1$ ; the lower status assumes it begins in the off state  $V_0, i = 0$ .

The investigation shows that the economic viability of the facility is sensitive to its ability to market its byproducts in addition to ethanol.

4.6. Future risk profile

In the next few figures, the distribution of profits, 95% value-at-risk (VaR) and conditional value-at-risk (CVaR) are provided followed by an investigation of the amount of time spent idle and operating at a loss. The  $VaR_\alpha$  of a project at given confidence level  $\alpha \in (0,1)$  is the smallest number  $\gamma$  such that the probability that the loss  $\Gamma$  exceeds

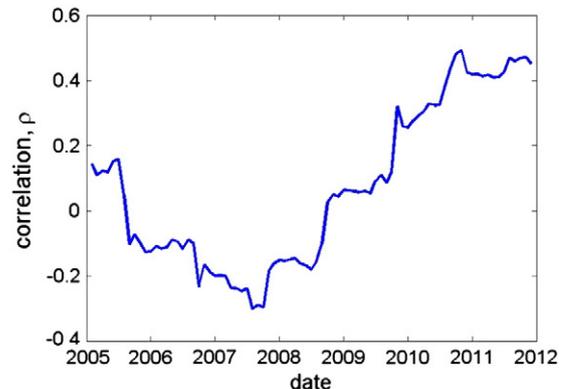


Fig. 8. The 3 year rolling correlation,  $\rho$ , over the 7 year period Jan/04–Dec/11 from the 10 year monthly price data, Jan/02–Dec/11. There is evidence of increased correlation in recent years which may be related to increased production and demand in corn ethanol.

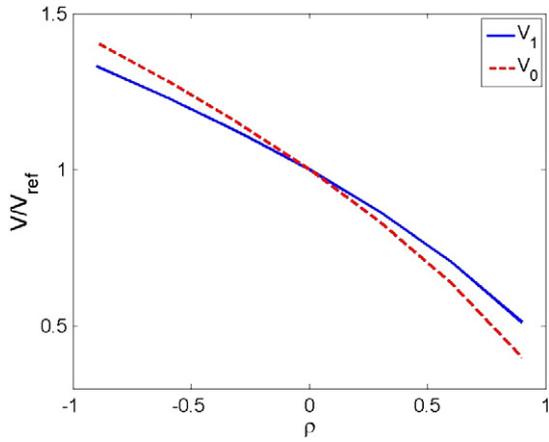


Fig. 9.  $V_1(l, c)/V_{1,ref}$  and  $V_0(l, c)/V_{0,ref}$  versus  $\rho$ . The initial ethanol  $l$  and corn  $c$  prices are  $(l, c) = (0.94, 1.90)$  and the reference value  $V_{i,ref}$  is taken at  $\rho = 0$ . The value is decreasing in  $\rho$ . All other parameters are as in Table 2.

$\gamma$  is at most  $(1 - \alpha)$ . The  $CVaR_{\alpha}$  is the expectation of this tail, i.e.  $CVaR_{\alpha} = E[X|X \leq VaR_{\alpha}]$ . The subsidy is taken to be zero,  $s = 0$ , for the investigation as it aims to investigate the cash flows going forward on a 10 year horizon.

The probability density function (PDF) of income assuming the project is immediately started,

$$Z = \int_t^T e^{-r(s-t)} f_{I_s} ds - \sum_{k=0}^T e^{-r(\tau_k-t)} D_{u_{k-1}, u_k} \tag{34}$$

is shown in Figs. 19 and 20. The investigation is performed at the Dec/11 price  $(L_t, C_t) = (2.49, 6.02)$ .

The large peaks in the distributions of incomes indicate the projects which remain idle for extended periods of time.

The experimental cumulative distribution function (CDF) of income and capitalized costs assuming the site is available on a 10 year horizon given the operator must first decide to initiate the project at optimal time  $\tau$ ,

$$M = \left\{ -e^{-r(\tau-t)} B + \int_{\tau}^T e^{-r(s-t)} f_{I_s} ds - \sum_{k=0}^T e^{-r(\tau_k-t)} D_{u_{k-1}, u_k} + Qe^{-r(T-t)} \right\} 1_{\tau < T} \tag{35}$$

is shown in Fig. 21. The jump in the CDF  $Prob(M \leq m)$  at zero indicates a large number of projects which are never optimally initiated. The large point mass at zero in Figs. 19–21 shows that many projects wait a very long time to begin or are in fact never initiated.

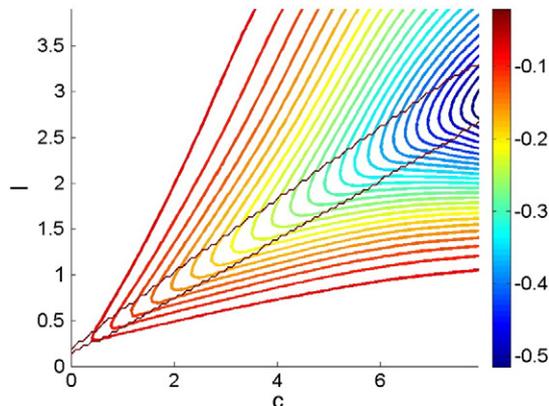


Fig. 10. The sensitivity  $\frac{\partial V_1}{\partial \rho}$  as a contour plot. Note that  $V_1$  is most sensitive to changes in  $\rho$  near the hysteresis zone. All parameters are as in Table 2.

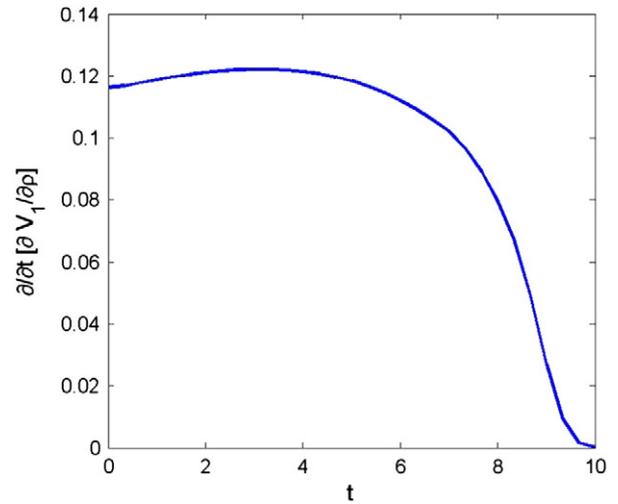


Fig. 11. The second-order sensitivity  $\frac{\partial^2 V_1}{\partial \rho^2}$  as a function of  $t$ . Note that  $V_0$  becomes more sensitive to changes in  $\rho$  as  $t$  increases. All parameters are as in Table 2.

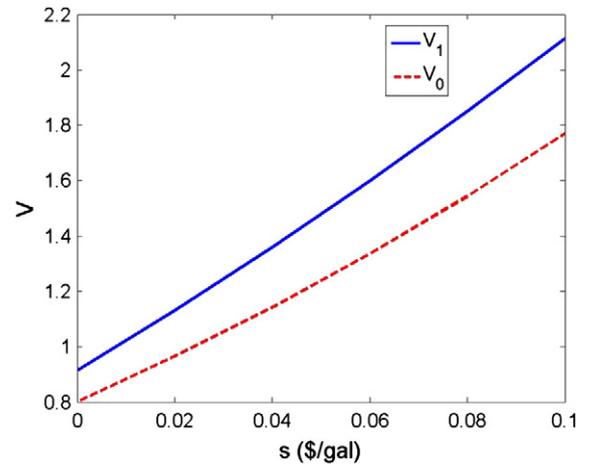


Fig. 12.  $V_1(l, c)$  and  $V_0(l, c)$  as a function of  $s$ . The value is increasing in  $s$ . All other parameters are as in Table 2.

The section concludes by investigating how increased correlation affects the following factors:  $VaR_{0.05}$ ,  $CVaR_{0.05}$ , fraction of time spent idle  $t_{idle}$ , the fraction of time spent operating at a loss  $t_{op loss}$ , and the

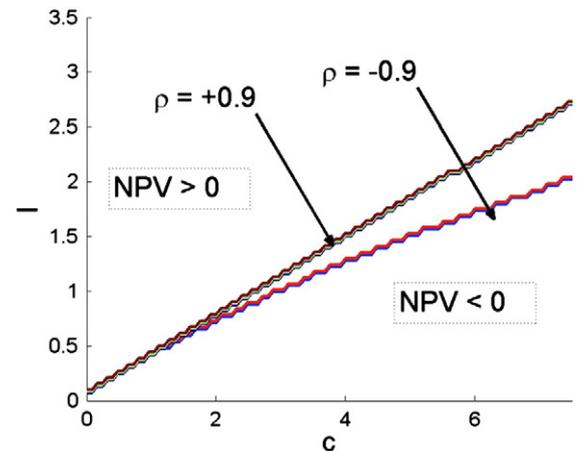


Fig. 13. NPV positivity regions at  $\rho = 0.9$  and  $\rho = -0.9$ . The boundaries denote the area over which projects are NPV positive. It is largest when  $\rho$  approaches  $-1$  and smallest when  $\rho$  approaches  $+1$ . The area bounded between the two regions indicates how the boundary (NPV positive set) decreases as  $\rho$  increases which implies that fewer projects are NPV positive. All other parameters are as in Table 2.

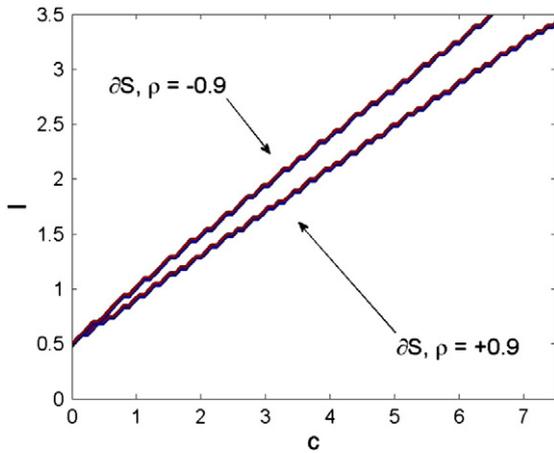


Fig. 14. The entry boundary  $\partial S$  as  $\rho$  increases. Note that the region over which we wait to invest,  $H$ , decreases as  $\rho$  increases. Compared with Fig. 13, the distance between deciding to invest and the region of NPV positivity shrinks as  $\rho$  increases. All other parameters are as in Table 2.

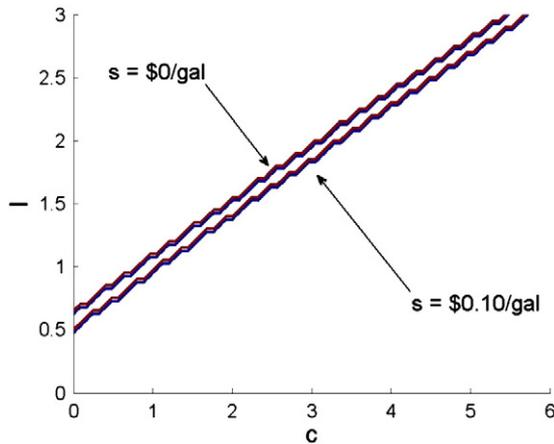


Fig. 15. The entry set boundary  $\partial S$  as  $K_1$  increases. Note that the region over which we wait to invest,  $H$ , increases as  $K_1$  increases. All other parameters are as in Table 2.

fraction time spent waiting to enter into the project  $\tau/T$ . These are summarized in Figs. 22 and 23 given the project is initiated optimally from a green field site.

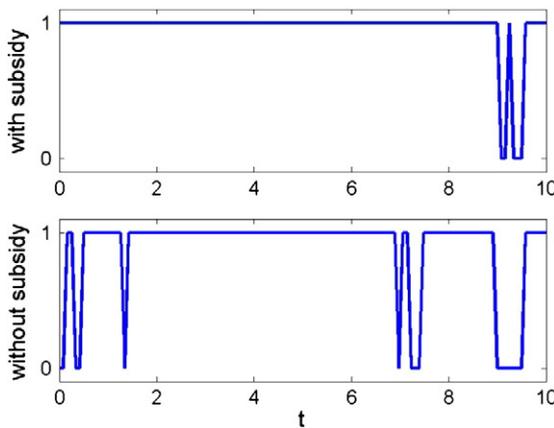


Fig. 16.  $1_{f_1(t,c)>0}$  with and without subsidy. Note that despite that there are moments when the facility is operating at a loss, the presence of switching costs filters the operating signal to be almost always on.

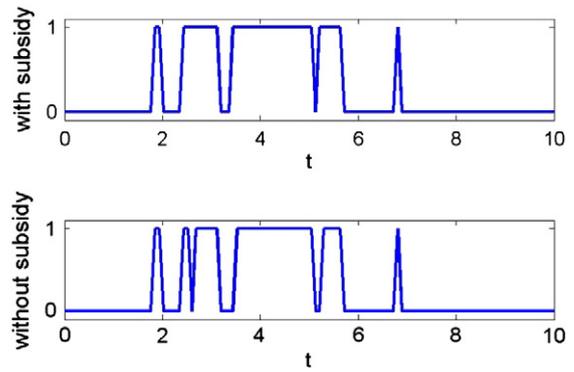


Fig. 17. Retrospective decision status whether to enter into the investment with and without subsidy. Note that it is optimal to wait nearly 2 years before initiating the projects. All other parameters are as in Table 2.

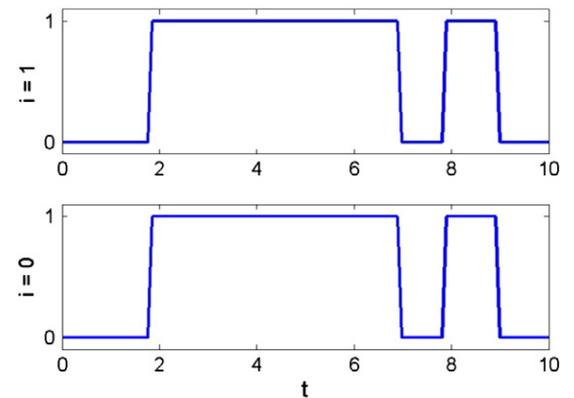


Fig. 18. Retrospective operating status of a facility with no marketable grains,  $\bar{A}_t = 0$ . Note that it is only optimal to be in production for roughly 60% of the time over the 10 year period. All other parameters are as in Table 2.

It can be observed from Figs. 22 and 23 that, as  $\rho$  increases, the value at risk of the project typically decreases. However, the project value also decreases supporting our earlier assertion that the optimal operating strategy becomes “less optimistic” in that the investor waits longer to enter. The amount of time spent idle or operating at a loss tends to decrease. This is expected since the investor has already waited until prices were more favorable before initially entering into the project.

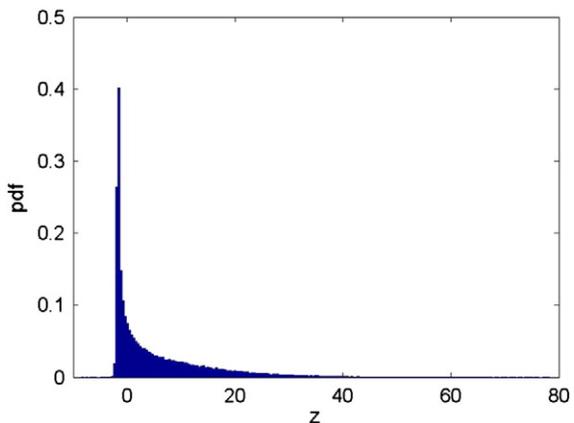


Fig. 19. Monte Carlo simulations of  $Z$  given  $i = 1$  following the optimal operating strategy.  $(L_n, C_i) = (2.49, 6.02)$  and all parameters are as in Table 2 with 100,000 simulations.

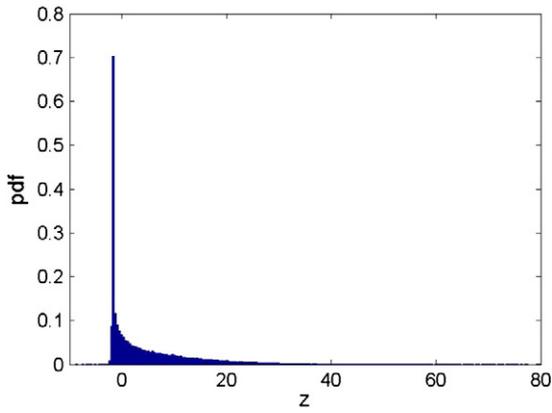


Fig. 20. Monte Carlo simulations of  $Z$  given  $i = 0$  following the optimal operating strategy.  $(L_b, C_t) = (2.49, 6.02)$  and all parameters are as in Table 2 with 100,000 simulations.

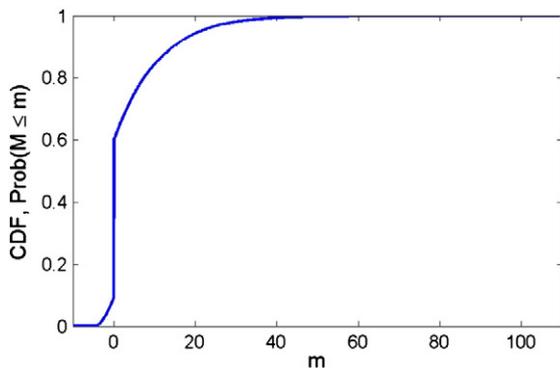


Fig. 21. Experimental CDF of  $M$  following the optimal operating strategy. Note the large point mass of projects which are never started or begin very late in the cycle.  $(L_b, C_t) = (2.49, 6.02)$  and all parameters are as in Table 2 with 100,000 simulations.

5. Discussion and conclusion

Our paper investigated the economic viability of a corn ethanol production facility using real option models. The results indicate that the viability of the project is sensitive to changes in correlation and subsidy policy along with the ability to market its byproducts.

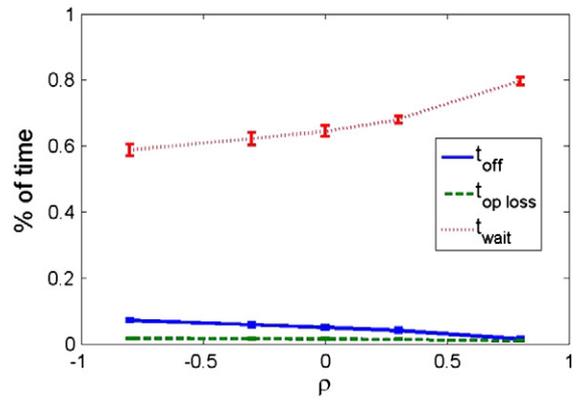


Fig. 23. Time spent idle, operating at a loss and waiting for entry as a function of  $\rho$ .  $(L_b, C_t) = (2.49, 6.02)$  and all parameters are as in Table 2 with 50,000 simulations. Error bars indicate 3 standard deviations.

5.1. Correlation

The investigations with the Margrabe exchange options showed that the option can lose over 70% of its value as the correlation increases from uncorrelated to nearly perfectly correlated,  $\rho \approx 0.9$  (Fig. 3). Further the complete model showed that given the deep in the money initial price at Jan/02, the facility can lose over 50% of its value as the prices become more correlated (Fig. 9). The contour plot of  $\frac{\partial V}{\partial \rho}$  (Fig. 10) showed that in the hysteresis zone, the facility is most sensitive to changes in correlation.

Our investigations using the infinite time horizon model indicated that as the correlation increased, the size of the hysteresis zone shrank (Fig. 5). This may indicate more certainty in the income cash flows but also indicates lowered expectation for value or prices rebounding favorably for the operator (Fig. 22). Additionally, our risk profile analysis indicated that in most cases, as correlation increases, the fraction of time spent waiting to start the project increases resulting in lowered productivity (Fig. 23).

From our investigation it is clear that, as correlation increases, the number of projects that are economically viable decreases. That is, the sets of initial prices for which the project is NPV positive shrinks as the prices become more correlated (Figs. 13–14). Thus fewer projects may be NPV positive, and hence not initiated, at any given time and price environment. Perhaps counterintuitively, the optimal price trigger at which to enter the project is in fact lowered as correlation increases but again this reflects lowered expectations for the project. The value

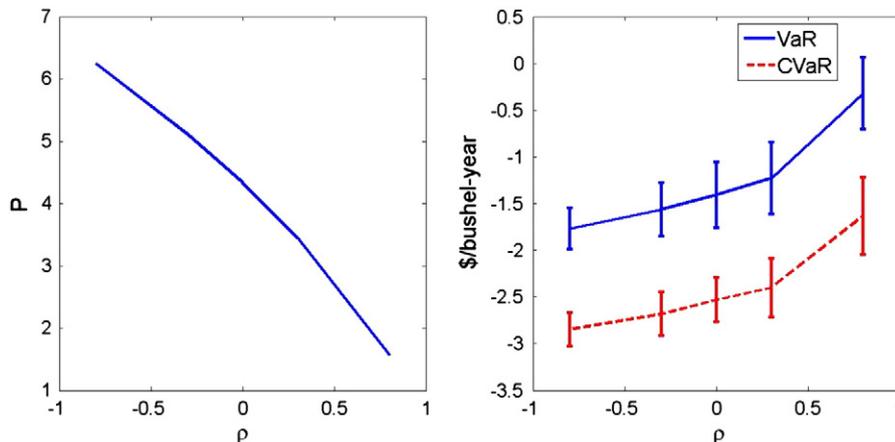


Fig. 22. Left: Expected value of investment  $P$ . Right: 5% VaR and CVaR of investment as a function of  $\rho$ .  $(L_b, C_t) = (2.49, 6.02)$  and all parameters are as in Table 2 with 50,000 simulations. Error bars indicate 3 standard deviations.

of waiting to invest is reduced since the optimal entry price trigger is moving closer to the region at which it is first NPV positive.

The risk profile investigation also yielded additional insight about the viability of the project. In particular, many of the projects are not NPV positive if the entry decision is made suboptimally. The PDF of potential realized profits shows that there is a large mass of risk-adjusted realizations that do not exceed the initial capitalized costs of construction. However if the option to enter the project is exercised optimally, the risk of losses is greatly reduced (Figs. 19–21).

### 5.2. Subsidy policy

As the Margrabe exchange option predicted, the value of the facility is semilinearly decreasing in  $s$ , the subsidy policy. Thus, for example, when the investment is deep in the money, the value of subsidy has a term proportional to  $s(T - t)$  in the absence of discounting. When the subsidy is removed, our investigation showed that the number of projects which were economically viable was reduced (Figs. 12 and 15). This was evidenced by a reduction in the set of prices for which the project was NPV positive.

Our retrospective analysis revealed an interesting fact about the optimal operating strategy for the facility: The subsidy had a minimal effect on the operating decisions regarding when to pause/resume production and to enter into the investment. With or without the subsidy, the decisions were nearly identical. Thus tax dollars are subsidizing a project that may in any case have been economically attractive, and investment capital is misappropriated from other possible projects. These numerical results indicate that the recent idling of many ethanol plants in 2013 may be the result of market factors as opposed to subsidy policy.

On the other hand, the subsidy may be successful in inducing ethanol production investment where none would otherwise exist. Although without the subsidy the facility would have historically been in production, the subsidy also reduces the operating risk. This has the effect of smoothing the distribution of income over the life of the project, reducing the presence of the distributional spike of projects which are never started. Thus a primary effect of the subsidy, and arguably a main goal, is to ameliorate the apparent risk profile of entering into the ethanol business; rather than to increase the value of the project or to influence operating decisions.

### 5.3. Efficiency of the facility

In the retrospective analysis, our paper showed that the success of the facility is contingent on its ability to market and sell its byproduct grains. It is possible that the facility may have difficulty collecting and marketing its distillers dried grain byproducts due to factors including its proximity to principle markets, its ability to collect and store the byproducts, and the grade or quality of the distillers dried grain byproducts. All of these factors will affect the price the operator can get and subsequently the value of the facility is strongly linked to the firm's ability to market its byproducts. In particular, the retrospective analysis showed that the facility would only be in production approximately 60% of the time if it were unable to market its grain.

Our investigation with simple Margrabe options showed that the loss in value is approximately semilinear in  $\kappa$ . Thus facility yield is also a key component to success for an ethanol facility; particularly in the presence of high corn prices.

### 5.4. Conclusion

Our paper provided an in-depth investigation of the retrospective and future economic viability of a typical North American corn ethanol production facility. It investigated the effects of ethanol policy manifested as increased price correlation due to increased demand for corn ethanol, as well as the direct effects of the subsidy on firms' operating decisions.

Our results show that the future viability of these facilities without the subsidy is still positive although with the subsidy, the effects of these risk factors are greatly reduced.

## Appendix A. Abandonment

In the above analysis the option to abandon was not considered. In this appendix we present a formalism for incorporating the option to abandon. We show that this omission is not material, at least in the parameter regimes considered in the current paper. Empirically abandonments are rarely observed in reality compared to the frequency of idling (NEB, 2013).

First, we observe that the option to abandon the facility can be considered an effective floor on the income of the facility. Consider for example an idle facility in the presence of very unfavorable ethanol and corn prices. It has the option to either idle at a loss for the foreseeable future or cut its losses and abandon, assuming the salvage value exceeds the expected accrued running costs or potential profits over the remaining facility life.

A facility can be abandoned from idle in which case the operator gets a salvage value  $F$ , or it can (in principle at least) be abandoned from the running state in which case a cost  $D$  somewhat less than  $D_{10}$  will be incurred. The total expected earnings over the life of the facility is

$$V_i(l, c, t) = \sup_{\tau, u, \theta} E \left[ \int_t^\theta e^{-r(s-t)} f_i(L_s, C_s) ds + \sum_{k=1}^n e^{-r(\tau_k-t)} D_{u_{k-1}, u_k} + 1_{\theta < \tau} e^{-r(\theta-t)} (F - 1_{u_\theta=1} D) + 1_{\theta \neq \tau} e^{-r(T-t)} Q \middle| (L_t, C_t, u_0) = (l, c, i) \right] \quad (\text{A.1})$$

where all notation is as previously defined. Here  $\theta$  is the optimal time to abandon whereupon the abandonment value is received. Given the facility is not abandoned before  $T$ , i.e.  $\theta \notin [t, T)$ , the salvage value  $Q$  is received at the end of the lease.

Dynamic programming reduces the problem to that of finding  $\tau$

$$V_i(l, c, t) = \sup_{\tau} E \left[ \int_t^\tau e^{-r(s-t)} f_i(L_s, C_s) ds + e^{-r(\tau-t)} \max \{ (V_j(L_\tau, C_\tau, \tau) - D_{ij}), F - 1_{i=1} D \} \middle| (L_t, C_t) = (l, c) \right] \quad (\text{A.2})$$

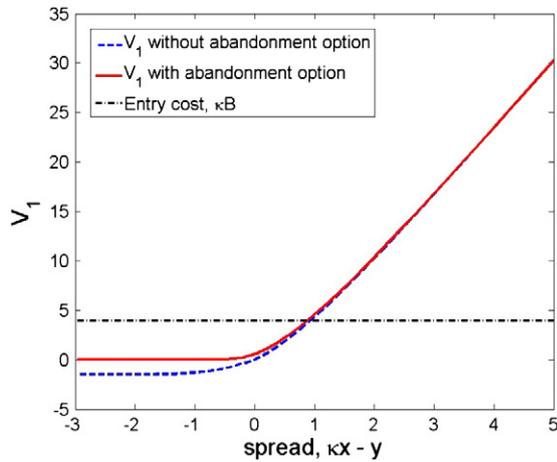
where  $\tau < T$ . The associated free boundary system is

$$\max \left[ \frac{\partial V_1}{\partial t} + L[V_1] + f_1(l, c, t) - rV_1, \max \{ (V_0 - D_{10}) - V_1, F - D \} \right] = 0, \quad (\text{A.3})$$

$$\max \left[ \frac{\partial V_0}{\partial t} + L[V_0] + f_0(l, c, t) - rV_0, \max \{ (V_1 - D_{01}) - V_0, F \} \right] = 0 \quad (\text{A.4})$$

with final conditions  $V_1(l, c, T) = V_0(l, c, T) = Q$ . Note that  $Q$  need not be the same as  $F$ , and in general will be larger, as  $Q$  incorporates the fact that the facility at the end of the lease may potentially be renovated and then continue to operate as a going concern. Accordingly it may have more value than just the scrapping and liquidation of its constituent parts.

The capitalized construction cost is much larger than the abandonment value,  $B > F$ , and thus the option to abandon does not materially affect the decision point to enter  $\partial S$ . In particular, since  $P \geq 0$  and  $F - B < 0$ , the decision to enter is never made at a point where abandonment would have occurred as per Eq. (24). For the parameters considered, at the lowest bound where entry to the investment may be considered (i.e. where  $V_1 = B$ ), the difference between the values  $V_1(l, c, t)$  with and without abandonment is very small. Fig. A.24 numerically illustrates this feature.



**Fig. A.24.**  $V_1(l, c, t)$  with and without the option to abandon as a function of the spread  $\kappa l - c$ . Near the lower limit value of NPV positive entry,  $V_1 = B$ , the difference is small. All parameters are as in Table 2 along with an abandonment value of  $F = 0.5Q$  and  $D = 0.75D_{10}$ .

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