Is Value Premium a Proxy for Time-Varying Investment Opportunities: Some Time Series Evidence

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Abstract

Recent authors argue that the value premium constructed from the cross-section of stocks is a proxy for investment opportunities. We show that this conjecture sheds light on the puzzling empirical risk-return tradeoff in the stock market across time. That is, in contrast with many early authors, we find that the stock market return is positively and significantly related to its conditional variance after controlling for its covariance with the value premium. The covariance, which is negatively correlated with stock variance, is positively and significantly priced as well. Therefore, by ignoring the effect of time-varying investment opportunities on the stock market return, the early specification might suffer from an omitted variables problem, which generates a downward bias in the estimate of the risk-return relation. Also, consistent with recent investment-based equilibrium models, we document a positive and significant relation between the value premium and its conditional variance over the post-1963 period. Overall, our empirical evidence suggests that the value premium might be a proxy for investment opportunities.

Keywords: ICAPM, value premium, stock return predictability, realized volatility, and GARCH.

JEL number: G1.

The capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) fails to explain the stock return data along two important dimensions. First, Fama and French (1993), for example, show that the CAPM does not account for the cross-section of stock returns, e.g., the value premium and the size premium.[1] Second, Campbell (1987), Whitelaw (1994), and Brandt and Kang (2004), among many others, find a weak or negative risk-return tradeoff in the stock market across time, in contrast with the positive relation stipulated by the CAPM. One possible explanation is that the CAPM assumption of constant investment opportunities is unrealistic because financial economists have documented mounting evidence of predictable variations in stock market returns and variance. Therefore, the CAPM-related anomalies suggest that the stock market might act as a hedge against changes in investment opportunities, as illustrated in Merton's (1973) intertemporal CAPM (ICAPM).

In particular, Fama and French (1995, 1996) argue that the value and size premiums move closely with investment opportunities and include them as additional risk factors in their three-factor model—perhaps one of the most influential and successful empirical asset pricing models. Consistent with Fama and French's conjecture, Liew and Vassalou (2000) find that the value premium forecasts output growth in many industrial countries. Also, Campbell and Vuolteenaho (2004), Brennan, Wang, and Xia (2004), and Petkova (2005), among others, show that the value premium is correlated with innovations in their measures of investment opportunities and the ICAPM appears to explain the cross-section of stock returns.[2] More importantly, recent authors, e.g., Gomes, Kogan, and Zhang (2003), and Zhang (2005), have

[1] The value premium is the return on a portfolio that is long in stocks with a high book-to-market value ratio (value stocks) and short in stocks with a low book-to-market value ratio (growth stocks). The size premium is the return on a portfolio that is long in stocks with small capitalizations and short in stocks with big capitalizations.
[2] Jagannathan and Wang (1996), among others, argue that the CAPM holds conditionally but not unconditionally, and Lettau and Ludvigson (2001a), Petkova and Zhang (2005), and Ang and Chen (2005) find that the conditional CAPM helps explain the value premium. Lewellen and Nagel (2005), however, are skeptical about the explanatory power of the conditional CAPM.

developed investment-based equilibrium models, which have fully specified structures that link the value premium to aggregate and firm-specific productivity shocks.

The main purpose of this paper is to investigate whether the value premium constructed from the cross-section of stocks sheds light on the on-going debate about the risk-return tradeoff in the stock market. If it is a proxy for investment opportunities, the conditional excess stock market return, $E_t(R_{t+1})$, is determined by its conditional variance, $\sigma^2_{M,t}$, and its conditional covariance with the value premium, $\sigma_{MH,t}$:

(1) $$E_t(R_{t+1}) = \gamma_M \sigma^2_{M,t} + \gamma_H \sigma_{MH,t},$$

where γ_M and γ_H are risk prices. The parameter γ_M is usually interpreted as the coefficient of relative risk aversion and thus should be positive. As we show below, γ_H also has an intuitive economic interpretation; in Campbell's (1993) ICAPM, it is also positive if γ_M is greater than 1. Similarly, the conditional value premium, $E_t(HML_{t+1})$, is determined by its conditional covariance with the stock market return, $\sigma_{MH,t}$, and its conditional variance, $\sigma^2_{H,t}$:

(2) $$E_t(HML_{t+1}) = \gamma_M \sigma_{MH,t} + \gamma_H \sigma^2_{H,t}.$$

To illustrate the main results, we first estimate equations (1) and (2) using realized variances and covariances constructed using daily return data, as advocated by Merton (1980) and Andersen, Bollerslev, Diebold, and Labys (2003), among others. Consistent with early authors, we find that realized stock market variance has little predictive power for returns over the quarterly sample period 1963:Q4 to 2002:Q4, the longest sample available to us when the paper was first written.[3] However, it becomes significantly positive after we include realized

[3] We focus on quarterly data rather than monthly data because Ghysels, Santa-Clara, and Valkanov (2005) argue that realized variance is a function of long distributed lags of squared past returns. Andersen, Bollerslev, Diebold, and Wu (2004) also advocate for using quarterly data rather than monthly data in a similar exercise. However, following

covariance between the stock return and the value premium, which is also significantly positive. Figure 1 shows that the realized covariance (solid line) is negatively correlated with the realized stock variance (dashed line). Given that both γ_M and γ_H are found to be positive, the estimate of γ_M is downward biased towards zero if we exclude the covariance term from equation (1). Therefore, by ignoring the effect of time-varying investment opportunities on stock market returns, the CAPM potentially suffers from an omitted variables problem.

Interestingly, we find that the value premium is positively and (marginally) significantly correlated with its conditional variance after controlling for its covariance with the stock return. This result appears to be consistent with Zhang's (2005) model, in which the value premium moves countercyclically because of its positive loadings on systematic risk. In the joint estimation of equations (1) and (2), we fail to reject the ICAPM restrictions of no constant terms and the same risk prices across assets at the conventional significance level. Moreover, as expected, imposing these restrictions helps estimate γ_M and γ_H more precisely; they are both found to be positive and highly significant. Overall, our results are consistent with the conjecture that the value premium is a priced risk factor because it is a proxy for investment opportunities.

The realized volatility model might not provide an efficient estimate for the conditional variances and covariances; for example, it does not adequately account for the effect of long distributed lags of returns (see footnote 3). For robustness, we also estimate a more elaborate bivariate GARCH-in-mean model and find very similar results for the post-1963 monthly sample. Two flexible specifications have been considered: The asymmetric BEKK (ABEKK) model by Engle and Kroner (1995) and the asymmetric dynamic covariance (ADC) model by

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the early authors, we use higher-frequency, e.g., monthly or weekly, data in the GARCH model. It should be noted that, unlike the simple realized volatility model, conditional variances and covariances are a weighted sum of squared past return innovations in the GARCH model.

Kroner and Ng (1998). For the ABEKK model, we fail to reject the ICAPM restrictions of no constant terms and the same risk prices across assets. In the preferred specification, γ_M is found to be positive and highly significant, with a point estimate of 4.6 and a standard error of 1.1. The point estimate falls comfortably within the plausible range of 1 to 10 for the coefficient of relative risk aversion, as advocated by Mehra and Prescott (1985), among others. The parameter γ_H is also found to be significantly positive, with a point estimate of 5.9 and a standard error of 1.8. We obtain very similar estimates using the more complicated ADC model; the data are not supportive of the ICAPM restrictions, however. Overall, the GARCH estimation also provides strong support for the conjecture that the value premium is a proxy for investment opportunities.

The GARCH model also allows us to extend the analysis to the early period, July 1926 to December 1962, over which we have only monthly data. This period is particularly interesting because Campbell and Vuolteenaho (2004), among others, find that the value premium does not pose a challenge to the CAPM over the early period, in contrast with the modern (post-1963) period. Campbell and Vuolteenaho argue that the difference reflects a structural break in the value premium: For various reasons, it did not co-move with innovations in investment opportunities in the early period as much as in the modern period. Their results have two important implications for our empirical specifications over the early period. First, the covariance with the value premium is not priced, or γ_H would be statistically insignificant. Second, the estimate of γ_M is downward biased towards zero because of the omitted variables problem. These conjectures are largely consistent with our empirical evidence. The parameter γ_H is found to be negligible and insignificant over the period July 1926 to December 1962; although γ_M is still significantly positive, its point estimate of 2.5 is substantially smaller than the point estimate of 4.6 obtained from the modern period.

Scruggs (1998) estimates a bivariate GARCH model using the long-term interest rate as a proxy for investment opportunities. However, his results are sensitive to the assumption of a constant correlation coefficient between stock market returns and the long-term interest rate (e.g., Scruggs and Glabadanidis [2003]). Guo and Whitelaw (2005) use the consumption-wealth ratio proposed by Lettau and Ludvigson (2001b) as a proxy for investment opportunities and find their results to be very similar to ours. For example, in their preferred specification (model 6 of Table 2), the point estimate of γ_M is 4.9 and the standard error is 2.1. The striking similarities between the two papers reflect the fact that, as we show in this paper, the consumption-wealth ratio and the covariance between stock market returns and the value premium capture some common predictable variations in stock market returns.

Although our empirical evidence is consistent with the hypothesis that the value premium is a measure of investment opportunities, we cannot completely rule out that alternative explanations exist. Nevertheless, our analysis raises the bar for the competing theories by showing a close link between time-series and cross-sectional stock return predictability. In particular, while the link is well established in Merton's ICAPM, it poses a challenge to the irrational pricing (e.g., Lakonishok, Shleifer, and Vishny [1994]) and data mining (e.g., MacKinlay [1995]) explanations for the value premium.

The remainder of the paper is organized as follows. We discuss the empirical specifications in Section I and present the estimation results of the realized volatility model in Section II. The bivariate GARCH model is discussed in Section III and some concluding remarks are offered in Section IV.

I. Empirical Specifications

In Merton's (1973) ICAPM, the conditional excess stock market return, $E_t(R_{t+1})$, is determined by its conditional variance, $\sigma^2_{M,t}$, and its covariances with the state variables, $\sigma_{MF,t}$:

(3) $$E_t(R_{t+1}) = [\frac{-J_{WW}W}{J_W}]\sigma^2_{M,t} + [\frac{-J_{WF}}{J_W}]\sigma_{MF,t},$$

where $J(W(t), F(t), t)$ is the indirect utility function with subscripts denoting partial derivatives, $W(t)$ is wealth, and $F(t)$ is a vector of state variables that describe investment opportunities. $\frac{-J_{WW}W}{J_W}$ is a measure of relative risk aversion, which is usually assumed to be constant. Following Scruggs (1998), we also assume that $\frac{-J_{WF}}{J_W}$ is a vector of constants.

In this paper, we assume that the value premium is a proxy for investment opportunities, i.e., $F_t = HML_t$. As mentioned in the introduction, we can motivate this specification using recent empirical evidence (e.g., Campbell and Vuolteenaho [2004], Brennan, Wang, and Xia [2004], and Petkova [2005]) and, especially, recent theoretical works (e.g., Gomes, Kogan, and Zhang [2003] and Zhang [2005]). Then it is straightforward to derive equation (1) from equation (3), with $\gamma_M = \frac{-J_{WW}W}{J_W}$ and $\gamma_H = \frac{-J_{WF}}{J_W}$. Equation (2) also follows directly from the ICAPM implication that the return on any asset is determined by its covariances with the stock market return and the state variables.

The intuition of our results can be easily illustrated using Campbell's (1993) ICAPM. If stock returns are predictable, Campbell and Shiller (1988) show that we can decompose the unexpected stock return into shocks to cash flows, $N_{CF,t+1}$, and shocks to discount rates, $N_{DR,t+1}$:

(4) $$R_{t+1} - E_t(R_{t+1}) = N_{CF,t+1} - N_{DR,t+1}.$$

Stock prices fall when there is a negative shock to cash flows or a positive shock to discount rates; however, the long-run effects of the two types of shocks are different. The positive discount-rate shock is associated with an improvement in investment opportunities, i.e., higher expected future stock returns. In contrast, investment opportunities do not change with the cash-flow shock. Therefore, discount rates are a measure of investment opportunities in Campbell's ICAPM, and the expected return on an asset, e.g., the market portfolio, is determined by its covariances with the stock market return (R_{t+1}) and the discount-rate shock ($-N_{DR,t+1}$):

(5) $$E_t(R_{t+1}) = \gamma_M \sigma^2_{M,t} + (\gamma_M - 1)\sigma_{M,DR,t},$$

where $\sigma_{M,DR,t}$ is the conditional covariance between R_{t+1} and $N_{DR,t+1}$. Substituting equation (4) into equation (5), we obtain

(6) $$E_t(R_{t+1}) = \gamma_M \sigma^2_{CF,t} + \sigma^2_{DR,t} - (\gamma_M + 1)\sigma_{CF,DR,t},$$

where $\sigma^2_{CF,t}$ is the conditional variance of the cash-flow shock, $\sigma^2_{DR,t}$ is the conditional variance of the discount-rate shock, and $\sigma_{CF,DR,t}$ is the conditional covariance between the cash-flow and discount-rate shocks. Equation (6) shows that the risk price of the cash-flow shock is equal to the coefficient of relative risk aversion, γ_M, and the risk price of the discount-rate shock is equal to 1. Therefore, if γ_M is greater than 1 (as found in this paper), the stock market risk is overstated in the CAPM, in which we assume that the risk prices are the same for both types of shocks. The second term in the RHS (right hand side) of equation (5) corrects for this difference in risk prices (note that $\sigma_{M,DR,t}$ is negative).

Similarly, Campbell and Vuolteenaho (2004) argue that the distinction between the cash-flow and the discount-rate shocks is important for understanding why the CAPM fails to explain the value premium in the post-1963 period. To illustrate their argument, we decompose the unexpected value premium into the discount-rate and the cash-flow shocks:

(7) $$HML_{t+1} - E_t(HML_{t+1}) = \beta_{H,DR} N_{DR,t+1} + \beta_{H,CF} N_{CF,t+1}.$$

Campbell and Vuolteenaho show that growth stocks tend to have a higher market beta than value stocks because the former has higher durations and thus is more vulnerable to discount-rate shocks (e.g., Cornell [1999]). That is, the value premium is closely correlated with the discount-rate shock, or $\beta_{H,DR}$ is high in equation (7). However, the value premium is positive because of its positive loadings on the cash-flow shock, which, as mentioned above, has a higher risk price than the discount-rate shock if γ_M is greater than 1. Therefore, the CAPM fails to explain the value premium because it assigns the same risk prices for the two types of shocks. As shown below, we confirm these results in our empirical analysis, although our approach is different from Campbell and Vuolteenaho. Note that equation (7) is also the main assumption adopted in Lettau and Wachter (2005), who develop an equilibrium model to explain the value premium.[4]

We assume that the high-minus-low portfolio is well diversified. This assumption seems reasonable because, to construct the value premium (as analyzed in this paper), Fama and French (1996) use all CRSP common stocks for which the accounting data from COMPUSTAT are available. Also, as shown in Table 1, the quarterly standard deviation of the value premium is 5.9%, compared with 8.7% for the stock market return.[5]

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[4] Our two-factor ICAPM nests the conditional CAPM by Zhang (2005) as a special case because Zhang assumes that the cash-flow and the discount-rate shocks are perfectly correlated with each other. However, it seems straightforward to extend his model to a two-factor setting by adding an independent discount-rate shock, as in Lettau and Wachter (2005). Also, the economic mechanism for a positive value premium uncovered in this paper is the same as that proposed by Zhang. That is, the positive value premium reflects its positive loadings on fundamental (i.e., cash flows) risk, especially during business downturns.

[5] If the value premium has an idiosyncratic component, the only modification that we need to make is to subtract the idiosyncratic volatility from the value premium volatility in equation (9). This modification implies a non-zero constant term in equation (9); however, as shown in our empirical analysis, we fail to reject the null hypothesis that the constant term is zero in most cases, suggesting that the idiosyncratic component might be small.

Substituting equation (7) into equation (5), we obtain

$$(8) \qquad E_t(R_{t+1}) = \gamma_M \sigma^2_{M,t} + (\gamma_M - 1)\frac{1}{\beta_{H,DR}}\sigma_{MH,t} - (\gamma_M - 1)\frac{\beta_{H,CF}}{\beta_{H,DR}}\sigma_{M,CF,t}.$$

If the value premium is perfectly correlated with the discount rate shock (i.e., $\beta_{H,CF}$ is equal to zero), equation (8) is exactly the same as equation (1). Moreover, as shown in Table 5 of Campbell and Vuolteenaho (2004), $\beta_{H,CF}$ is much smaller than $\beta_{H,DR}$ in the post-1963 sample. Therefore, omitting the third term in the RHS of equation (8) is likely to have a relatively small effect. In this case, equation (1) holds approximately. Similarly, it is straightforward to show:

$$(9) \qquad E_t(HML_{t+1}) = \gamma_M \sigma_{MH,t} + (\gamma_M - 1)\frac{1}{\beta_{H,DR}}\sigma^2_{H,t} - (\gamma_M - 1)\frac{\beta_{H,CF}}{\beta_{H,DR}}\sigma_{H,CF,t}.$$

Equation (9) is (approximately) equivalent to equation (2) if $\frac{\beta_{H,CF}}{\beta_{H,DR}}$ is negligible. Note that the second term in the RHS of equation (9) reflects the fact that the discount-rate shock has a lower risk price than the cash-flow shock. Therefore, our main empirical specification is:

$$(10) \qquad \begin{aligned} E_t(R_{t+1}) &= \alpha_M + \gamma_{MM}\sigma^2_{M,t} + \gamma_{HM}\sigma_{MH,t} \\ E_t(HML_{t+1}) &= \alpha_H + \gamma_{MH}\sigma_{MH,t} + \gamma_{HH}\sigma^2_{H,t} \end{aligned},$$

and the ICAPM requires $\alpha_M = \alpha_H = 0$, $\gamma_{MM} = \gamma_{MH} = \gamma_M$, and $\gamma_{HM} = \gamma_{HH} = \gamma_H = (\gamma_M - 1)\frac{1}{\beta_{H,DR}}$.

Since $\beta_{H,DR}$ is found to be positive in Campbell and Vuolteenaho, γ_H is positive if and only if γ_M is greater than 1. Also, γ_H is constant if $\beta_{H,DR}$ is constant.

Equation (10) holds only approximately if the value premium is not perfectly correlated with the discount-rate shock. To partially address this issue, we make two additional simplifying assumptions: The two types of shocks in equation (4) are uncorrelated; and the betas in equation (7) are constant.[6] Therefore, equations (6) and (9) imply

$$(11) \quad E_t\begin{bmatrix} R_{t+1} \\ HML_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_M & 1 \\ \gamma_M \beta_{H,CF} & -\beta_{H,DR} \end{bmatrix} \begin{bmatrix} \sigma^2_{CF,t} \\ \sigma^2_{DR,t} \end{bmatrix}.$$

Equations (4) and (7) imply:

$$(12) \quad \begin{bmatrix} \sigma^2_{M,t} \\ \sigma^2_{H,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \beta^2_{H,CF} & \beta^2_{H,DR} \end{bmatrix} \begin{bmatrix} \sigma^2_{CF,t} \\ \sigma^2_{DR,t} \end{bmatrix}.$$

If the matrix $\begin{bmatrix} 1 & 1 \\ \beta^2_{H,CF} & \beta^2_{H,DR} \end{bmatrix}$ is not singular, equations (11) and (12) imply

$$(13) \quad E_t\begin{bmatrix} R_{t+1} \\ HML_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_M & 1 \\ \gamma_M \beta_{H,CF} & -\beta_{H,DR} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \beta^2_{H,CF} & \beta^2_{H,DR} \end{bmatrix}^{-1} \begin{bmatrix} \sigma^2_{M,t} \\ \sigma^2_{H,t} \end{bmatrix}.$$

Equation (13) shows that, under moderate conditions, the conditional stock market return and the value premium are linear functions of their conditional variances. As shown below, we find that, as expected, equation (13) appears to provide a better description for the data than does equation (10).

The ICAPM suggests that we should use variables that forecast stock market returns as proxies for investment opportunities; however, it provides little guidance for the choice of the stock return predictors. Also, innovations in the state variables are not directly observable and Campbell and Vuolteenaho (2004), for example, must rely on some admittedly ad hoc assumptions to identify them. Therefore, although more defendable than an empirical APT model, the empirical ICAPM is also potentially sensitive to variations in these specifications (see, e.g., Chen [2003] and Chen and Zhao [2005]). In contrast, by directly linking the value

[6] The first assumption is consistent with the empirical evidence by Campbell and Vuolteenaho (2004), who also assume that betas are constant over various samples.

premium to investment opportunities, as suggested by recent investment-based equilibrium models (e.g., Zhang [2005]), we do not need to identify the priced state variables and their innovations. Our specification thus is complementary to that adopted by Campbell and Vuolteenaho (2004), among others; also, it allows us to address some related issues, e.g., the risk-return tradeoff in the stock market across time.

As mentioned in footnote 3, we use quarterly data for the realized volatility model and monthly data for the GARCH model. For comparison, we conduct Monte Carlo simulations by assuming that daily returns follow a bivariate ABEKK process, as estimated from daily data. Our preliminary results suggest that, while both models are reasonably reliable for the sample size used in this paper, the point estimates of risk prices obtained from the GARCH model appear to be closer to the parameters set in the simulation than those obtained from the realized volatility model. Of course, although the simulation results are suggestive, they should be interpreted with caution because we do not know the true data-generating process. In particular, French, Schwert, and Stambaugh (1987) argue that while full information likelihood estimators such as GARCH are potentially more efficient, they are not as robust to model misspecification as instrumental variable models. Also, the realized volatility model allows us to compare the value premium with other potential measures of investment opportunities proposed by early authors. Therefore, in this paper, we follow French, Schwert, and Stambaugh and present estimation results obtained from both the realized volatility model and the GARCH model.

II. Realized Volatility Model

We obtain daily and monthly data of the Fama and French three factors from Professor Ken French at Dartmouth College.[7] When the paper was first written, daily data were available over the period July 1963 to December 2002 and monthly data were available over the period July 1926 to February 2004. Following Merton (1980), among many others, we use the sum of the squared daily returns in a quarter as a measure of realized variance. The realized covariance is the sum of the cross-product of daily excess stock market returns and the value premium.[8] Note that we use simple returns in the empirical analysis and construct quarterly simple returns by aggregating monthly returns through compounding.

In Figure 1, we plot realized stock market variance, $v_{M,t}^2$ (dashed line), along with realized covariance between the stock market return and the value premium, $v_{MH,t}$ (solid line). Note that $v_{M,t}^2$ rose dramatically during the 1987 stock market crash but reverted to the normal level shortly after. Many authors, e.g., Schwert (1990), argue that the 1987 crash is unusual in many ways. Unless otherwise indicated, we replace realized variance for 1987:Q4 with the second largest observation in our sample, as in Campbell, Lettau, Malkiel, and Xu (2001). $v_{MH,t}$ is almost always negative and exhibits substantial fluctuations across time: Its absolute value tends to be relatively high just before or during business recessions (dated by the National Bureau of Economic Research), as denoted by the shaded areas. As mentioned above, the two variables in Figure 1 usually move in opposite directions. Similarly, as shown in Figure 2, realized variance of the value premium, $v_{H,t}^2$ (solid line), is also negatively related to $v_{MH,t}$ (dashed line). Lastly, Figure 3 shows that, while realized variance of the stock market return (dashed line) is closely related to realized variance of the value premium (solid line), the

[7] We downloaded the data from his website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.

correlation is not perfect possibly because the stock market return and the value premium have different loadings on the cash-flow and discount-rate shocks.

Table 1 presents summary statistics for the excess stock market return and the value premium as well as their realized variances and covariance over the period 1963:Q4 to 2002:Q4. The excess stock market return, R_t, is negatively related to the value premium, HML_t, with a correlation coefficient of –0.48. Consistent with Figures 1 to 3, panel A also shows that $v_{M,t}^2$, $v_{H,t}^2$, and $v_{MH,t}$ are closely related to each other; however, the correlation is far from being perfect. Lastly, panel B shows that the three variance and covariance measures exhibit substantial persistence: The autocorrelation coefficients are 0.56, 0.72, and 0.55 for $v_{M,t}^2$, $v_{H,t}^2$, and $v_{MH,t}$, respectively. Therefore, realized variance and covariance are good predictors of their future levels.

A. Estimation of Merton's ICAPM

We can rewrite equation (10) in the realized return form and use realized variances and covariances as proxies for their conditional values:

$$(14)\qquad \begin{aligned} R_{t+1} &= \alpha_M + \gamma_{MM} v_{M,t}^2 + \gamma_{HM} v_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH} v_{MH,t} + \gamma_{HH} v_{H,t}^2 + \varepsilon_{H,t+1} \end{aligned},$$

where $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the stock return and the value premium, respectively. We present the GMM (generalized methods of moments) estimation results in Table 2.

We first discuss the stock return equation, as reported in panel A of Table 2. Row 1 replicates the familiar result that realized stock market variance, $v_{M,t}^2$, has weak forecasting

[8] As in French, Schwert, and Stambaugh (1987), we find essentially the same results by correcting the serial

power for the excess stock market return, R_{t+1}: Its coefficient is positive but only marginally significant, with an adjusted R-squared of 1.5%. However, it remains positive and becomes significant at the 1% level after we control for the realized covariance, $v_{MH,t}$ (row 2). Interestingly, the effect of $v_{MH,t}$ is also significantly positive, and the adjusted R-squared increases to 4.6%. Our results suggest a classic omitted variables problem. Note that $v^2_{M,t}$ and $v_{MH,t}$ are both positively related to R_{t+1}, although they are negatively related to each other (Table 1). Therefore, the estimate of γ_{MM} is biased downward towards zero if we do not control for $v_{MH,t}$, as in row 1.

Similarly, as shown in row 1 of panel B, Table 2, realized variance of the value premium, $v^2_{H,t}$, is not significantly correlated with the one-quarter-ahead value premium, HML_{t+1}. However, its coefficient becomes marginally significant after we control for the covariance, $v_{MH,t}$ (row 2). Given that $v^2_{H,t}$ and $v_{MH,t}$ are both positively related to the value premium (row 2) but negatively correlated with each other (Table 1), these results again suggest an omitted variables problem.

In row 3 of Table 2, we estimate the two equations jointly: We use a constant, $v^2_{M,t}$, and $v_{MH,t}$ as instrumental variables for the stock return equation and a constant, $v^2_{H,t}$, and $v_{MH,t}$ for the value premium equation. Thus the equation system is just-identified and the point estimates are identical to those reported in row 2. Note that from row 3 on, we report the R-squared rather than the adjusted R-squared (as in rows 1 and 2) in the column under $\overline{R}^2$. We impose the restrictions that the constant terms are zero in both equations and report the results in row 4. The

correlation in daily return data. For brevity, these results are not reported here but are available upon request.

restrictions can be tested using Hansen's (1982) J-test, which has a chi-squared distribution with 2 degrees of freedom. The J-test statistic is 1.14, indicating that the restrictions are not rejected at the 50% significance level. As expected, imposing ICAPM restrictions also improves the estimation efficiency, and, in particular, the effect of $v_{H,t}^2$ now becomes significant at the 5% level in the value premium equation. In row 5, we impose the restrictions that the risk prices are equal across assets; they are not rejected and the risk prices are found to be significantly positive. Lastly, we impose the restrictions of no intercepts and the equal risk prices across assets and report the results in row 6. Again, the restrictions are not rejected and the risk prices are found to be positive and highly significant. Overall, our results suggest that, as argued by Campbell and Vuolteenaho (2004) and Zhang (2005), the value premium is a proxy for investment opportunities.

Early authors, e.g., Fama and French (1989) and Campbell (1987), find that the dividend yield, the default premium, the term premium, and the stochastically detrended risk-free rate forecast stock market returns. Ferson and Harvey (1999) show that these variables also have predictive power for the value premium. One possibility is that they co-move with realized variance and covariance in equation (14) at the business-cycle frequency. To address this issue, we include them as instrumental variables, in addition to those used in row 6 of Table 2. Row 7 shows that the model is not rejected at the 10% significance level, indicating that the stock return predictability documented by early authors is indeed consistent with the ICAPM.

Recently, Lettau and Ludvigson (2001b) argue that the consumption-wealth ratio, CAY_t, is a strong predictor of stock market returns. If we also add CAY_t to our instrumental variable set (row 8, Table 2), the model is not rejected only at the 1% significance level; the other results, however, are very similar to those reported in rows 6 and 7. Therefore, again, our results suggest that the value premium reflects intertemporal pricing, although it might be a noisier measure of investment opportunities than other stock return predictors proposed in the literature.

As shown in Figures 1 through 3, stock volatility rose to a high level in the last few years of our sample, during which stock prices first increased sharply and then collapsed with the burst of the technology bubble. To investigate whether this seemingly unusual episode has any special effect on our results, we also analyze a shorter sample spanning the period 1963:Q4 to 1997:Q4 and report the results in rows 9 and 10 of Table 2, which have the same specifications as those in rows 7 and 8, respectively. The results are very similar to those obtained using the full sample.

B. Forecasting Stock Market Returns

Using a relatively recent sample, Lettau and Ludvigson (2001b) show that the stochastically detrended risk-free rate, $RREL_t$, and the consumption-wealth ratio, CAY_t, are strong predictors of stock market returns. These variables also subsume the information content of many commonly used predictors, including the term premium, the default premium, and the dividend yield. According to Merton's ICAPM, e.g., equation (3), $RREL_t$ and CAY_t forecast returns because they co-move with either stock market variance or the covariance between the stock market return and investment opportunities. Motivated by this intuition, Guo and Whitelaw (2005) find a positive risk-return relation after controlling for $RREL_t$ and CAY_t as proxies for the covariance term. Also, Guo and Savickas (2005) find that a measure of value-weighted idiosyncratic volatility, IV_t, also forecasts stock market returns when combined with stock market variance. Guo and Savickas interpret IV_t as a proxy for realized variance of an omitted risk factor, for example, realized variance of the value premium in equation (13). Therefore, if the value premium is indeed a priced risk factor, the forecasting abilities of $v_{MH,t}$ might be related to those of $RREL_t$, CAY_t, and IV_t. We address this issue in Table 3.

As shown in row 1 of Table 3, the forecasting power of $v^2_{M,t}$ and $v_{MH,t}$ is essentially unchanged in the presence of $RREL_t$, of which the coefficient is significantly negative, as found by many early authors. In contrast, $v_{MH,t}$ loses the predictive power after we control for CAY_t (row 2) or IV_t (row 3), while the effect of $v^2_{M,t}$ remains positive and highly significant.

The relatively weak forecasting power of $v_{MH,t}$ (when CAY_t or IV_t is included) might reflect the fact that the value premium is not perfectly correlated with the shock to discount rates or investment opportunities. For example, the third term in the RHS of equation (8) is not negligible. To address this issue, we forecast stock market returns using $v^2_{H,t}$ instead of $v_{MH,t}$, as suggested by equation (13). As shown in row 4 of Table 3, the coefficients of $v^2_{M,t}$ and $v^2_{H,t}$ are both highly significant, with the adjusted R-squared of 7.8%—a noticeable increase from 4.6% reported in row 2 of Table 2. Nevertheless, the effect of $v^2_{H,t}$ again becomes insignificant at the 5% level after we control for CAY_t (row 6) or IV_t (row 7). Overall, our results indicate that the value premium appears to contain important information about investment opportunities. Also note that the close link between the value premium and CAY_t and IV_t strongly suggests that their forecasting power for stock returns reflects intertemporal pricing and should not be fully attributed to data mining or irrational pricing.[9]

[9] Various factors might explain the relatively weaker performance of the value premium. For example, Fama and French (1996) argue that the value premium is only an empirical risk factor and does not explain the momentum effect documented by Jegadeesh and Titman (1993). Also, in Zhang's (2005) model, the conditional value premium is a nonlinear function of the priced state variable (Figure 4) and thus the correlation between the two might not be perfect. In contrast, CAY_t, for example, might be a better measure of investment opportunities because it is motivated directly from economic theories.

In row 4 of Table 3, the coefficient of $v_{H,t}^2$ is negative, while the coefficient of $v_{M,t}^2$ is positive. This is because $v_{H,t}^2$ forecasts stock market returns due to its negative co-movements with $v_{MH,t}$ (Table 1), which in turn is positively related to stock market returns. This result helps explain the seemingly puzzling negative relation between IV_t and stock returns documented by Guo and Savickas (2005). Our result suggests that the negative relation reflects the fact that, as advocated by Guo and Savickas, IV_t is a proxy for the variance of an omitted risk factor.[10]

C. Out-of-Sample Forecast

Bossaerts and Hillion (1999) and Goyal and Welch (2003), among others, have challenged the robustness of the in-sample evidence of stock return predictability. In particular, they show that many commonly used forecasting variables have negligible out-of-sample predictive power. Ferson, Sarkissian, and Simin (2003), among others, have also cautioned about the spurious regression and data mining. To address these issues, we conduct the out-of-sample analysis in this subsection. In particular, we use three statistics to compare the out-of-sample performance of the model using $v_{H,t}^2$ and $v_{M,t}^2$ as predictors (as in row 4, Table 3) with a benchmark of constant excess stock returns. First is the mean-squared forecasting error (MSE) ratio. Second is Clark and McCracken's (2001) encompassing test (ENC-NEW); this test compares the null hypothesis that the benchmark model incorporates all the information about the next quarter's excess stock market return against the alternative hypothesis that our forecasting variables provide additional information. Third is McCracken's (1999) test of equal forecast accuracy (MSE-F). In the MSE-F test, the null hypothesis is that the benchmark model

[10] We show the link between the idiosyncratic volatility and the volatility of the omitted risk factor in an appendix, which is available upon request.

has an MSE less than or equal to that of the augmented model compared against the alternative hypothesis that the augmented model has a smaller MSE. Clark and McCracken (2001) show that the latter two tests have the best overall power and size properties among a variety of tests proposed in the literature.

We report the results of the out-of-sample forecast tests in Table 4. As in Lettau and Ludvigson (2001b), we use the first third of the observations for the initial in-sample estimation and form the out-of-sample forecast recursively in the remaining sample.[11] That is, we use the observations over the period 1963:Q4 to 1976:Q4 to make the forecast for 1977:Q1 and update the sample to 1977:Q1 to forecast the return for 1977:Q2 and so forth. The column MSE_A / MSE_B is the MSE ratio of the augmented model to that of the benchmark model. The column Asy. CV reports the 95% critical value from the asymptotic distribution provided by Clark and McCracken (2001) and McCracken (1999). The column BS. CV is the 95% critical value obtained from bootstrapping, as in Lettau and Ludvigson (2001b). Consistent with the in-sample regression results, we find that the augmented model has a smaller MSE than the benchmark model of constant stock returns. More importantly, both the ENC-NEW and MSE-F tests reject the null hypothesis that $v_{H,t}^2$ and $v_{M,t}^2$ provide no information about future stock returns at the 5% significance level using both the asymptotic and bootstrapping critical values.

For robustness, in Figure 4 we plot the recursive MSE ratio (of the augmented model to the benchmark model of constant returns) through time. The horizontal axis denotes the starting forecast date; for example, the value corresponding to March 1977 is the MSE ratio over the forecast period 1977:Q1 to 2002:Q4. We choose the range 1977:Q1 to 1997:Q4 for the starting

[11] The ratio of the number of observations used in the out-of-sample forecast to that used in the initial in-sample regression is a crucial parameter for the critical values of the last two tests. Clark and McCraken (2001) provide the critical values for only selected values of this ratio. Following Lettau and Ludvigson, we set it to be 2 so that we can obtain the critical value from Clark and McCraken. Nevertheless, we find very similar results using various samples.

forecast date; therefore, we utilize at least 20 observations in the calculation of MSE. As shown in Figure 4, the MSE ratio is always less than 1, indicating that the out-of-sample predictive power of $v_{H,t}^2$ and $v_{M,t}^2$ is not sensitive to any particular sample period.

D. Forecasting the Value Premium

Consistent with equation (2), $v_{H,t}^2$ has some forecasting power for the value premium when combined with $v_{MH,t}$ (row 2 of Table 2). This result indicates that predictable variations in the value premium documented by some early authors (e.g., Ferson and Harvey [1999] and Guo and Savickas [2005]) might be consistent with intertemporal pricing. To address this issue, in Table 5, we compare the forecasting power of realized variances and covariance with alternative measures of investment opportunities, namely, $RREL_t$, CAY_t, and IV_t.[12] As shown in row 1, the effect of $v_{H,t}^2$ remains positive and marginally significant after controlling for $RREL_t$. However, the effect of $v_{H,t}^2$ becomes insignificant after we control for CAY_t (row 2) and IV_t (row 3), both of which have insignificant effects as well.

Row 4 of Table 5 presents the regression results using $v_{M,t}^2$ instead of $v_{MH,t}$ in the forecasting equation, as suggested by equation (13). Consistent with the results reported in Table 3 for stock market returns, the alternative specification appears to provide a better fit for the value premium as well. Now the effect of $v_{H,t}^2$ is positive and significant at the 5% level; the effect of $v_{M,t}^2$ is negative and also significant at the 5% level. Also, the adjusted R-squared is 4.8%, which is noticeably higher than the 3.6% reported in row 2 of Table 2. The coefficient of

$v_{M,t}^2$ is negative because of its negative correlation with $v_{MH,t}$ (Table 1), which in turn is positively correlated with the value premium.

The forecasting power of $v_{H,t}^2$ (as in row 4 of Table 5) is very similar to that of IV_t, as reported by Guo and Savickas (2005). These authors show that IV_t and $v_{M,t}^2$ jointly have strong predictive power for the value premium; moreover, while $v_{M,t}^2$ is negatively correlated with the one-quarter-ahead value premium, the relation is positive for IV_t. To formally address this issue, we also include IV_t in the forecasting equation, together with $v_{H,t}^2$ and $v_{M,t}^2$. As shown in row 7, while the coefficient of $v_{M,t}^2$ remains significantly negative, the coefficients of both IV_t and $v_{H,t}^2$ become insignificant, indicating that the two variables indeed capture common variations in the value premium. This result should not be too surprising because, as argued by Guo and Savickas (2005), by construction, IV_t is a proxy for realized variance of an omitted risk factor (e.g., the realized variance of the value premium, as in equation (13)). The positive relation between the value premium and IV_t or $v_{H,t}^2$ is also consistent with recent investment-based equilibrium models (e.g., Gomes, Kogan, and Zhang [2003] and Zhang [2005]), in which these variables move countercyclically. However, in contrast with IV_t, controlling for $RREL_t$ (row 5) or CAY_t (row 6) does not affect our results in any qualitative manner.

E. Robustness

[12] The term premium, the default premium, and the dividend yield (as used by Ferson and Harvey [1999]) do not provide additional information about the future value premium, and including them does not change our results in any qualitative manner. To conserve space, these results are not reported here but are available upon request.

Fama and French construct the value premium using value weighting. In their Table 5, Campbell and Vuolteenaho (2004) find that growth stocks have a disproportionately higher discount-rate beta than value stocks do in the post-1963 sample, even after controlling for size. Therefore, we expect to find very similar results using the value premium constructed with both small and big stocks. To investigate this issue, we obtain from Kenneth French the daily return data for six portfolios, which are the intersection of two independent sorts: Size (big and small) and the book-to-market value ratio (high, median, and low). As shown in Table 6, we find essentially the same results using realized variance of the value premium constructed from small and large stocks. We also find that $v_{M,t}^2$ and $v_{H,t}^2$ forecast the Fama and French 25 portfolios sorted by size and the book-to-market value ratio; for brevity, these results are not reported here but are available upon request.

III. Multivariate GARCH Model

Many authors, e.g., Christensen and Prabhala (1998), Fleming (1998), and Guo and Whitelaw (2005), find that realized variance is not an efficient measure of conditional variance. Given the appealing economic intuition, our results are unlikely to be affected by this problem in any qualitative manner. The point estimates of the ICAPM parameters, however, could be biased. For example, as reported in Table 2, the coefficient of relative risk aversion, γ_M, has a point estimate above 8 and in some specifications exceeds 10, the upper bound of the plausible range considered by Mehra and Prescott (1985). To address this issue, in this section we estimate equation (10) using more elaborate bivariate GARCH models, which might provide a better measure for the conditional second moments than the simple realized volatility model. Again, we rewrite equation (10) in the realized return form:

$$
\begin{aligned}
R_{t+1} &= \alpha_R + \gamma_{MM}\sigma^2_{M,t} + \gamma_{HM}\sigma_{MH,t} + \varepsilon_{M,t+1} \\
HML_{t+1} &= \alpha_H + \gamma_{MH}\sigma_{MH,t} + \gamma_{HH}\sigma^2_{H,t} + \varepsilon_{H,t+1}
\end{aligned} \quad , \tag{15}
$$

where $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to stock market returns and the value premium, respectively.

We use the ADC model proposed by Kroner and Ng (1998). These authors show that it is very flexible in describing the dynamic of covariance terms because it nests several commonly used multivariate GARCH models. In the ADC model, the dynamic of variances and covariances is governed by the following equations:

$$
\begin{aligned}
&\sigma^2_{M,t} = \theta_{MM,t+1} \\
&\sigma^2_{H,t} = \theta_{HH,t+1} \\
&\sigma_{MH,t} = \rho_{MH}\sqrt{\theta_{MM,t+1}\theta_{HH,t+1}} + \phi_{MH}\theta_{MH,t+1} \\
&\theta_{ij,t+1} = \omega_{ij} + b_i' H_t b_j + a_i' \begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{H,t} \end{bmatrix} \begin{bmatrix} \varepsilon_{M,t} & \varepsilon_{H,t} \end{bmatrix} a_j + g_i' \begin{bmatrix} \eta_{M,t} \\ \eta_{H,t} \end{bmatrix} \begin{bmatrix} \eta_{M,t} & \eta_{H,t} \end{bmatrix} g_j ,\ i,\, j \in (H, M)
\end{aligned} \quad , \tag{16}
$$

where H_t is the conditional variance-covariance matrix:

$$
H_t = \begin{bmatrix} h_{MM,t} & h_{MH,t} \\ h_{MH,t} & h_{HH,t} \end{bmatrix} = \begin{bmatrix} \sigma^2_{M,t-1} & \sigma_{MH,t-1} \\ \sigma_{MH,t-1} & \sigma^2_{H,t-1} \end{bmatrix}. \tag{17}
$$

Glosten, Jagannathan, and Runkle (1993), among many others, find that a negative return shock leads to a higher subsequent volatility than a positive return shock of the same magnitude does. This asymmetric effect is captured by the term $\begin{bmatrix} \eta_{M,t} \\ \eta_{H,t} \end{bmatrix} = \begin{bmatrix} \max[0, -\varepsilon_{M,t}] \\ \max[0, -\varepsilon_{H,t}] \end{bmatrix}$ in equation (16). As pointed out by Kroner and Ng (1998), this term also allows for an asymmetric effect in covariance. That is, an increase in the information flow following bad news can cause not only the asymmetric effect in its own variance, but also in the covariance due to a change in the relative rate of information flow across different types of firms or market segments. ρ_{MH} and ϕ_{MH} are scalar parameters and the other parameters can be written in matrix forms:

$$W = C'C = \begin{bmatrix} \omega_{MM} & \omega_{MH} \\ \omega_{MH} & \omega_{HH} \end{bmatrix}, \quad A = \{a_M, a_H\} = \begin{bmatrix} a_{MM} & a_{MH} \\ a_{HM} & a_{HH} \end{bmatrix}$$

(18) ,

$$B = \{b_M, b_H\} = \begin{bmatrix} b_{MM} & b_{MH} \\ b_{MH} & b_{HH} \end{bmatrix}, \quad G = \{g_M, g_H\} = \begin{bmatrix} g_{MM} & g_{MH} \\ g_{HM} & g_{HH} \end{bmatrix}$$

where W is positive definite and C is a 2 × 2 symmetric matrix. Note that our notations in equation (18) reflect the fact that matrixes W and B are symmetric but matrixes A and G are not.

The ADC model is appealing because it nests several popular multivariate GARCH specifications. In particular, Kroner and Ng (1998) show that, if matrixes A and B are diagonal and ϕ_{MH} is equal to 0, it becomes the asymmetric version of the constant conditional correlation model, as used by Scruggs (1998), for example. If ρ_{MH} is equal to 0 and ϕ_{MH} is equal to 1, then the ADC model reduces to the asymmetric version of the popular BEKK model proposed by Engle and Kroner (1995), which, as we show below, seems to apply in this study.

We estimate the model using the quasi-maximum likelihood (QML) method. Bollerslev and Woodridge (1992) show that QML parameter estimates can be consistent, even though the conditional log-likelihood function assumes normality while stock returns are known to be skewed and leptokurtic. Also, as discussed below, we find very similar results using the maximum likelihood estimation (MLE) method by assuming a *t* distribution or a normal distribution.

Given a sample of T observations of the return vector, the parameters of the bivariate GARCH model are estimated by maximizing the conditional log-likelihood function:

$$L = \sum_{t=1}^{T} l_t(P) = \sum_{t=1}^{T} (-\log(2\pi) - 0.5\log|H_t| - 0.5\varepsilon_t' H_t^{-1}\varepsilon_t)\,,$$

where P denotes the vector of all the parameters to be estimated. Nonlinear optimization techniques are used to calculate the maximum likelihood estimates based on the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm.

The ADC model should be estimated under some parameter restrictions to ensure the positive definite covariance matrix. It is possible to impose the constraint $|\rho_{mv}|+|\phi_{mv}|<1$ in the model. To serve a similar purpose, Scruggs and Glabadanidis (2003) propose to penalize the likelihood function whenever the covariance matrix is not positive definite, which we followed in this study. While such treatment might lose the continuity of the likelihood function, it gains the ability to impose a less restrictive constraint and avoid the possibility of a non-positive definite covariance matrix. Also, imposing a penalty in the likelihood function often results in a function with multiple local optima. In this case, it is important to restart the optimization routine at several different starting points to ensure that the estimated parameters correspond to the global maximum of the likelihood function. All our results are tested for robustness using different starting values in the maximization of the likelihood function.

A. Data

Campbell and Vuolteenaho (2004) find a structural break in the components of the value premium's market beta. In the post-1963 period, the value premium has disproportionately large loadings on the shock to discount rates—a measure of investment opportunities in their ICAPM. It, however, has a large discount-rate beta as well as a large cash-flow beta in the early (pre-1963) period, indicating that the value premium is a poor proxy for investment opportunities and thus is well explained by the CAPM in that period. An immediate implication for our exercise is that the value premium should be a priced risk factor in the modern period but not in the early period. Therefore, as in Campbell and Vuolteenaho (2004), we estimate the GARCH model using two monthly subsamples—the modern period January 1963 to February 2004 and the early period July 1926 to December 1962—although we focus mainly on the modern period because its results are directly comparable with those of the realized volatility model. Consistent with Campbell and Vuolteenaho, we confirm that while the CAPM explains the value premium well in the early sample, an ICAPM is needed for the modern sample (also see Petkova and Zhang [2005]).

Table 7 provides summary statistics of the excess stock market return and the value premium in percentage over the period January 1963 to February 2004, the main focus of our analysis. Consistent with quarterly data in Table 1, the two variables are negatively correlated, with a correlation coefficient of –0.32. The Ljung-Box test indicates that the value premium is serially correlated.

B. Model Selection Tests

Kroner and Ng (1998), among others, argue that choosing a parsimonious GARCH specification is important for the asset pricing tests because they critically depend on the covariance matrix estimates. In fact, the ADC model was originally proposed to facilitate the model selection (Kroner and Ng, 1998, p. 833). A parsimonious data-determined model is desirable because the number of observations is limited but a large amount of the data is required to yield precise estimates of GARCH models. Hence, it is important in this study to impose statistically acceptable constraints and reduce the redundant parameters.

The model selection test follows the general-to-specific approach. Similar to Scruggs (1998) and Scruggs and Glabadanidis (2003), we first look at the second-moment modeling and then the first-moment modeling and report the results in Table 8. We first conduct a likelihood ratio test to examine the importance of allowing for interrelationships between the two conditional second moments of the stock market return and the value premium. The null model is the pooling of two univariate GARCH specifications, and the alternative model is the full-fledged bivariate ADC model. As shown in panel A of Table 8, the test statistic is 112.93, which follows a chi-squared distribution with 10 degrees of freedom. Therefore, the null is strongly rejected, suggesting that we need to simultaneously estimate two conditional second moments and allow for interactions between them.

Second, we test whether the more restrictive, and yet quite general, ABEKK model provides a sufficient description for the dynamics of stock returns. This requires a joint test of $\rho_{MH} = 0$ and $\phi_{MH} = 1$ in equation (16). As shown in Panel B of Table 8, the likelihood ratio test statistic is 1.30 and we fail to reject the null hypothesis. Given the fact that the ADC model involves more parameters and thus is more difficult to estimate, we hereafter focus on the ABEKK model in the remaining discussion, although we find similar results using the ADC model. Imposing such constraints should also help improve the efficiency of the estimation and reduce the standard errors of parameter estimates.

Third, we would like to examine whether conditional second moments respond asymmetrically to past return shocks. We test this hypothesis by comparing a symmetric BEKK model of conditional second moments with the asymmetric BEKK model. The null hypothesis of symmetry is strongly rejected, as shown in Panel C of Table 8.

We then turn to the model selection test on the first-moment modeling, which focuses on the intertemporal relation between risk and expected returns. The first test of interest is whether the excess stock market return and the value premium are not related to the time-varying conditional second moments, or $\gamma_{MM} = \gamma_{MH} = \gamma_{HM} = \gamma_{HH} = 0$ in the ABEKK model (Panel D of Table 8). The four restrictions are jointly rejected at the conventional significance level, indicating that conditional variance and covariance terms are significant determinants of the excess stock market return and the value premium. Merton's ICAPM also dictates that the constant terms should be zero or $\alpha_R = \alpha_H = 0$ in equation (15). As shown in panel E, we fail to reject these restrictions at the 30% significance level. Lastly, the theory also requires that the risk prices should be equal across assets or $\gamma_{MM} = \gamma_{MH}$ and $\gamma_{HM} = \gamma_{HH}$. We test this hypothesis jointly with the hypothesis of no constant terms $\alpha_E = \alpha_H = 0$, although the results are essentially the same without the bundling. Again, panel F shows that these restrictions are not rejected at the conventional significance level. Therefore, consistent with the results obtained from the realized volatility model, Merton's ICAPM also provides a good description of the data, using the ABEKK model.

C. Model Estimation

Table 9 presents the estimation results of the mean equations. We use the percentage return in the estimation; to make them comparable with the results in Table 2, we scale the constant terms by 1/100 and the slope parameters by 100.

For comparison with the early literature, we first report in panel A of Table 9 the estimation results of pooling univariate asymmetric GARCH model, i.e., we restrict the interaction terms between the stock market return and the value premium to be zero in equation (16). Note that the covariance term does not show up in both equations because it is restricted to be zero. For the excess stock market return equation, the conditional mean is positively related to the conditional variance with a point estimate of 1.21; however, the relation is not statistically significant at the conventional level. Similarly, we find a positive but insignificant risk-return relation for the value premium. Nevertheless, as in Table 2, such a result is tenable because the specification potentially suffers from an omitted variables problem, which we discuss next.

Panel B of Table 9 presents the estimation results using the ABEKK model. In the unrestricted specification (row 2), all the parameters are insignificant. Given that the slope parameters are jointly significant (panel D of Table 8), this result indicates that our estimation is not efficient. One way to address this issue, as we have learned from the realized volatility model reported in Table 2, is to impose the restrictions dictated by Merton's ICAPM. When the mean equations do not include the constant terms, the price of stock market risk becomes significant in the stock market return equation, with a point estimate of 5.9 and a standard error of 2.01 (row 3). Moreover, four slope parameters in the mean equations are now statistically significant at the 1% level, after we impose further restrictions that the risk prices are equal across assets, as shown in row 4. Given that these restrictions are not rejected by the data (Table 8), our results clearly demonstrate the importance of imposing the parameter constraints dictated by the theory to improve the estimation efficiency, which is particularly relevant in our nonlinear estimation with a relatively small number of observations.

The point estimates of risk prices in row 4 of Table 9, the preferred specification, are reasonable. The price of stock market risk, γ_M, which is usually interpreted as a measure of relative risk aversion, has a point estimate of 4.64 and a standard error of 1.12. It thus falls comfortably within the plausible range from 1 to 10, as advocated by Mehra and Prescott (1985), and many other financial economists. Interestingly, it is strikingly similar to the point estimate of 4.93 reported by Guo and Whitelaw (2005), who use the consumption-wealth ratio as a proxy for the covariance between the stock market return and investment opportunities. This is possibly because, as shown in Table 3, the two variables capture some common variations of stock market returns in the realized volatility model. The risk price of the value premium, γ_H, has a point estimate of 5.86 and a standard error of 1.77. As shown in equations (8) and (9), γ_H is equal to $\frac{(\gamma_M - 1)}{\beta_{H,DR}}$; our point estimates thus imply that $\beta_{H,DR}$ is equal to 0.62. This result is in line with the estimate reported in Table 5 of Campbell and Vuolteenaho (2004) for the post-1963 period and thus confirms that the value premium is indeed highly correlated with the discount-rate shock.

Figure 5 plots the fitted values of conditional stock market variance (dashed line) and covariance between the stock market return and the value premium (solid line) from the estimation reported in row 4 of Table 9. The pattern is very similar to that of realized variance and covariance in Figure 1. In particular, the two components of conditional stock market returns are negatively correlated, with a correlation coefficient of –0.72. Given that they are both positively related to stock market returns, our results again suggest an omitted variables problem in the univariate GARCH model, which has been commonly used in this literature. Figure 6 illustrates a negative relation between conditional variance of the value premium (solid line) and covariance between the value premium and the stock market return (dashed line). The pattern also explains that we fail to uncover a significant risk price for the value premium in the univariate GARCH model (row 1 of Table 9) because of the omitted variables problem.

Figure 7 shows that the conditional variances of stock market return and the value premium tend to move in the same directions; however, the correlation is not perfect, with a correlation coefficient of 0.53. Lastly, Figure 8 shows that there are also substantial variations in the conditional coefficient of correlation between the stock market return and the value premium.

The latter result confirms the finding of Scruggs and Glabadanidis (2003) that it is important to allow for a time-varying correlation coefficient in the ICAPM estimation.

Table 10 presents the parameter estimates of the benchmark ABEKK model, in which we impose all the ICAPM restrictions. Panels A and B report the estimates of the mean equations, which are the same as those in row 4 of Table 9. Panels C, D, and E show that most elements of each of the parameter matrices *W*, *A*, *B*, and *G* of the three conditional second moments are significant. These results are consistent with the existence of a time-varying variance-covariance matrix. Although the parameter estimates in general are not easy to interpret, they can still shed some light on the volatility process in a bivariate framework. Specifically, as shown by Engle and Kroner (1995, p. 127), the significance of b_{MM}, a_{MM}, b_{HH}, and a_{HH} indicates the GARCH effects on the second moments for both the stock market return and value stock portfolio return. The significance of a_{MH} and a_{HM} verifies the existence of cross-asset volatility spillovers. These parameter estimates suggest that the absolute size of return shocks originating in one asset, as measured by the squared value of lagged unpredictable returns, transmits to the current period's conditional volatility in the other asset. Another channel of such volatility transmission is nonexistent (i.e., an insignificant b_{MH}); that is, the conditional variance in one asset is not dependent on that of the other asset in the last period. Finally, the significance of g_{MM} and g_{HH} suggests the asymmetric effect in volatility in both markets. However, there is no evidence for the asymmetric effect in the conditional covariance, since neither g_{MH} nor g_{HM} is significant.

In panel C of Table 7 we report the mean of fitted values of conditional variances and covariance based on the estimation results reported in Table 10. They are very similar to the unconditional variance-covariance matrix of the excess stock market return and the value premium, as reported in panel B of Table 7.

D. Robustness Checks

As mentioned above, Campbell and Vuolteenaho (2004) show that the value premium is more sensitive to the discount-rate shock than to the cash-flow shock in the modern (post-1963) period but not the early (pre-1963) period. Their results have two implications for the empirical ICAPM specification of equation (10) for the early period. First, we expect that the price of the value premium is negligible and statistically insignificant. Second, it suffers from an omitted variable problem; therefore, the price of stock market risk is likely to be biased downward towards zero. As shown in row 5 of Table 9, these conjectures are strongly supported by the estimation results of the ABEKK model for the period July 1926 to December 1962. The risk price associated with the value premium has a negligible point estimate of 0.006, with a standard error of 1.66, indicating that it is not significant at any conventional level. The price of stock market risk is again statistically significant; nevertheless, its point estimate of 2.47 is substantially smaller than the point estimate of 4.64 obtained from the modern period, as shown in row 4 of Table 9. We also report the results using the full sample in row 6. Again, we find that the value premium risk is not priced but the price of the market risk is significantly positive. Given the structural break documented by Campbell and Vuolteenaho, this result should be interpreted with caution.

Although we concentrate on a restricted ABEKK specification in the previous discussion, it is worthwhile to note that we find similar results using the ADC model (as shown in panel C of Table 9). In the unrestricted model (row 7), we find that the risk prices are all positive though statistically insignificant. Interestingly, the risk prices become significant at the 1% level after we impose the ICAPM restrictions (row 8 of Table 9). Moreover, the point estimates are very similar to those obtained using the ABEKK model (as shown in row 4 of Table 9). However, the likelihood ratio test rejects the ICAPM restrictions for the ADC model possibly because the value premium is a noisy proxy for investment opportunities.

We also estimate the restricted ABEKK model using the MLE method by assuming a *t* distribution and a normal distribution for the modern sample and report the main results in panel D of Table 9. For the *t* distribution, the degree of freedom of the distribution has a point estimate of 7.85, which is also highly significant. This result is consistent with the general belief that the distribution of stock returns is characterized by heavy tails. Both the magnitudes and the significance levels of most parameter estimates are close to those under the normality assumption with only a few exceptions (mainly in matrix *G*). In particular, as shown in row 9, the two coefficients of interest, γ_M and γ_H, are estimated to be 4.79 and 6.26, with a standard error of 1.06 and 1.66, respectively (row 9). For the normal distribution, the parameters γ_M and γ_H have the same point estimates as those in row 4, with a standard error of 1.11 and 1.85, respectively. Therefore, our results appear to be robust to alternative estimation methods.

Lastly, we also repeat the above analysis using weekly and daily data. Again, our main finding that the covariances with the stock market return and the value premium carry a positive and significant risk premium holds well in the modern period under various specifications. For brevity, these results are not reported here but are available upon request.

E. Diagnostics Tests

To evaluate the adequacy of the benchmark ABEKK model reported in Table 10, we conduct several specification tests on the standardized residuals ($\hat{\varepsilon}_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}, \quad i = M, H$) and standardized products of residuals ($\hat{\varepsilon}_{i,t} \hat{\varepsilon}_{j,t} = \varepsilon_{i,t}\varepsilon_{j,t} / h_{ij,t}, \quad i = M, H$). Specifically, we examine some moment conditions required for the consistency of QML estimates. Panel A of Table 11 shows that the two mean standardized residuals are not significantly different from zero. The evidence is somewhat mixed with testing the null hypothesis that the mean of the products of the residuals is 1, however. The null cannot be rejected for $\hat{\varepsilon}_{M,t}\hat{\varepsilon}_{M,t}$ and $\hat{\varepsilon}_{H,t}\hat{\varepsilon}_{H,t}$ but can be rejected for the cross-product, $\hat{\varepsilon}_{M,t}\hat{\varepsilon}_{H,t}$. We also note that the skewness and kurtosis for the standardized residuals is much lower than the skweness and kurtosis for the value premium but not for the stock market return. Panel B of Table 11 summarizes the Ljung-Box test for autocorrelation in the estimated residual series. The autocorrelation is still present in the residuals of HML equation (recall that the original HML value series contains autocorrelation). Overall, these results indicate that, while the model provides a reasonable description of the data, there is still room for improvement.

IV. Conclusion

In this paper, we estimate a variant of Merton's (1973) ICAPM using the value premium as a proxy for time-varying investment opportunities. In contrast with many early authors, we uncover a positive and significant risk-return tradeoff after controlling for covariance between the stock market return and the value premium. We also document a novel empirical result—a significantly positive relation between conditional mean and variance of the value premium using the ICAPM specification. These results, which are consistent with Campbell and Vuolteenaho (2004), Brennan, Wang, and Xia (2004), Petkova (2005), and Zhang (2005), suggest that the value premium might not be fully attributed to irrational pricing or data mining.

Our results shed light on the on-going debate about stock market return predictability in time-series data, which has been widely documented in past decades. It is consistent with three hypotheses: irrational pricing, data mining, and the time-varying risk premium. While it has been difficult to distinguish between these alternatives, some recent authors, e.g., Bossaerts and Hillion (1999), Goyal and Welch (2003), and Ferson, Sarkissian, and Simin (2003), are more inclined to subscribe to the data mining explanation for at least two reasons. First, the forecasting variables are empirically motivated. Second, their forecasting performance varies substantially across different sample periods. This study allows us to make three statements here. First, existing economic theories have provided guidance for identifying predictive variables, for example, conditional variances and covariances of stock market returns and state variables in Merton's ICAPM. Second, despite its simplicity, our analysis shows that the theoretically motivated variables forecast stock market returns in sample and out of sample. Third, many financial variables forecast stock returns because of their co-movements with conditional variances and covariances of stock market returns and other risk factors.

Jagannathan and Wang (1996), among others, argue that the CAPM holds conditionally but not unconditionally because of time-varying betas and the stock market risk premium. Given that the second moments of asset returns exhibit substantial variations across time, we confirm the importance of using conditional asset pricing models. Nevertheless, our results also indicate that the conditional CAPM does not adequately explain the post-1963 data because the value premium is found to be a priced risk factor.

Recent authors provide tentative explanations why stock market returns are predictable, which is not explained in Merton's ICAPM. For example, in the habit formation model by Campbell and Cochrane (1999), expected returns change over time because of time-varying relative risk aversion. In contrast, in the limited stock market participation model by Guo (2004), investors require a liquidity premium to hold stocks, in addition to the risk premium in the standard model because of various market frictions. Interestingly, Guo shows that the liquidity premium, which is closely tracked by the aggregate consumption-wealth ratio, could be negatively related to the risk premium, i.e., conditional stock market variance, although each of them individually is positively related to excess stock market returns. Guo's model appears to provide a reasonable explanation for the results documented in this paper. For example, as shown in Table 3, the forecasting power of various measures of the second risk factor—e.g., covariance between the stock market return and the value premium, variance of the value premium, and the idiosyncratic volatility—is closely related to that of the aggregate consumption-wealth ratio. A further investigation of these results seems to be warranted and we leave it for future research.

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Table 1 Summary Statistics of Quarterly Return Data

Note: We report summary statistics for the excess stock market return, R_t; the value premium, HML_t; realized stock market variance, $v^2_{M,t}$; realized variance of the value premium, $v^2_{H,t}$; and realized covariance between the stock market return and the value premium, $v_{MH,t}$. The sample spans the period 1963:Q4 to 2002:Q4.

	R_t	HML_t	$v^2_{M,t}$	$v^2_{H,t}$	$v_{MH,t}$
Panel A: Correlation Matrix					
R_t	1.000				
HML_t	-0.483	1.000			
$v^2_{M,t}$	-0.366	0.092	1.000		
$v^2_{H,t}$	-0.276	0.285	0.697	1.000	
$v_{MH,t}$	0.362	-0.189	-0.820	-0.929	1.000
Panel B: Univariate Statistics					
Mean	0.013	0.012	0.005	0.001	-0.002
Standard Deviation	0.087	0.059	0.004	0.002	0.002
Autocorrelation	0.033	0.138	0.555	0.719	0.551

Table 2 Merton's ICAPM: Realized Volatility Model

Note: We report the estimation results of the Merton's ICAPM of equation (14) using the GMM:

$$(14)\quad \begin{aligned} R_{t+1} &= \alpha_M + \gamma_{MM} v^2_{M,t} + \gamma_{HM} v_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH} v_{MH,t} + \gamma_{HH} v^2_{H,t} + \varepsilon_{H,t+1} \end{aligned},$$

where R_{t+1} is the excess stock market return; HML_{t+1} is the value premium; $v^2_{M,t}$ is realized stock market variance; $v_{MH,t}$ is realized covariance between the stock market return and the value premium; $v^2_{H,t}$ is realized variance of the value premium; and $\varepsilon_{M,t+1}$ and $\varepsilon_{M,t+1}$ are shocks to the stock market return and the value premium, respectively. Unless otherwise indicated, we use the quarterly sample spanning the period 1963:Q4 to 2002:Q4. The heteroskadesticity-corrected standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. In the column under $\overline{R}^2$, the adjusted R-squared is reported in rows 1 to 2 and the R-squared for the other rows. The two equations are estimated separately in rows 1 and 2 and jointly in the other rows. The system is just identified in row 3: We use a constant, $v^2_{M,t}$, and $v_{MH,t}$ as instrumental variables for the stock market return equation and use a constant, $v^2_{H,t}$, and $v_{MH,t}$ for the value premium equation. We impose the restriction of zero intercept in row 4, the restriction of the same risk prices in row 5, and both restrictions in rows 6 to 8. We use the same instrumental variables in rows 4 to 6 as in row 3. We also include the default premium, the term premium, the stochastically detrended risk-free rate, and the dividend yield as instrumental variables in row 7. Row 8 also includes the consumption-wealth ratio by Lettau and Ludvigson (2001b) as an instrumental variable. We report Hansen's (1982) J-test in the column under J-Test. Rows 9 and 10 have the same specifications as rows 7 and 8, respectively, but are estimated for the sample period 1963:Q4 to 1997:Q4.

	Panel A. Stock Market Returns				Panel B. the Value Premium				J-Test
	α_R	γ_{MM}	γ_{HM}	$\overline{R}^2$	α_H	γ_{MH}	γ_{HH}	$\overline{R}^2$	
1	-0.000	2.907*		0.015	0.005		4.733	0.017	
	(0.009)	(1.730)			(0.005)		(3.359)		
2	-0.007	8.762***	12.827**	0.046	0.005	10.662	17.160*	0.036	
	(-.009)	(3.096)	(5.825)		(0.005)	(7.519)	(9.928)		
3	-0.007	8.762***	12.827**	0.058	0.005	10.662	17.160*	0.048	
	(0.009)	(3.096)	(5.825)		(0.005)	(7.519)	(9.928)		
4		8.112***	13.131**	0.057		11.794	20.033**	0.048	X(2)=1.14
		(2.291)	(5.088)			(7.486)	(9.306)		(0.57)
5	-0.006	9.220***	14.277***	0.058	0.006	9.220***	14.277***	0.048	X(2)=0.51
	(0.009)	(2.969)	(5.393)		(0.005)	(2.969)	(5.393)		(0.78)
6		9.200***	16.028***	0.054		9.200***	16.028***	0.047	X(4)=2.06
		(2.009)	(3.941)			(2.009)	(3.941)		(0.72)
7		8.476***	15.463***	0.051		8.476***	15.463***	0.046	X(12)=18.17
		(1.937)	(3.821)			(1.937)	(3.821)		(0.11)
8		9.090***	16.338***	0.052		9.090***	16.338***	0.047	X(14)=27.00
		(1.965)	(2.830)			(1.965)	(2.830)		(0.02)
9		11.766***	19.110***	0.025		11.766***	19.110***	0.008	X(12)=18.61
		(2.239)	(5.046)			(2.239)	(5.046)		(0.098)
10		13.264***	19.557***	0.033		13.264***	19.557***	0.00	X(14)=25.82
		(2.209)	(4.892)			(2.209)	(4.892)		(0.03)

Table 3 Forecasting Quarterly Excess Stock Market Returns

Note: We report the OLS regression results of forecasting one-quarter-ahead excess stock market returns using some predetermined variables over the period 1963:Q4 to 2002:Q4. The heteroskadesticity-corrected standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. $v_{M,t}^2$ is realized stock market variance; $v_{H,t}^2$ is realized variance of the value premium; $v_{MH,t}$ is realized covariance between the stock market return and the value premium; $RREL_t$ is the stochastically detrended risk-free rate; CAY_t is the consumption-wealth ratio proposed by Lettau and Ludvigson (2001b); and IV_t is a measure of idiosyncratic volatility used in Guo and Savickas (2005).

	$v_{M,t}^2$	$v_{MH,t}$	$v_{H,t}^2$	$RREL_t$	CAY_t	IV_t	$\bar{R}^2$
1	7.839***	11.623**		-4.757**			0.067
	(2.968)	(5.698)		(2.280)			
2	9.433***	7.800			2.313***		0.147
	(3.051)	(5.931)			(0.509)		
3	11.015***	2.679				-3.360***	0.100
	(3.015)	(6.664)				(0.896)	
4	8.168***		-17.038***				0.078
	(2.193)		(5.203)				
5	7.567***		-16.325***	-4.834**			0.100
	(2.128)		(5.075)	(2.225)			
6	9.007***		-10.874*		2.139***		0.159
	(2.172)		(5.558)		(0.539)		
7	10.511***		-5.234			-2.923**	0.102
	(2.275)		(8.367)			(1.283)	

Table 4 Out of Sample Forecast Test

Note: We assume that excess stock market returns are constant in the benchmark model and augment the benchmark model with realized stock market variance, $v^2_{M,t}$, and realized variance of the value premium, $v^2_{H,t}$. We report three out-of-sample forecast tests: (1) the mean-squared forecasting error (MSE) ratio of the augmented model to the benchmark model, MSE_A / MSE_B; (2) the encompassing test ENC-NEW developed by Clark and McCracken (2001); and (3) the equal forecast accuracy test MSE-F developed by McCracken (1999). ENC-NEW tests the null hypothesis that the benchmark model encompasses all the relevant information about the next quarter's excess stock market return against the alternative hypothesis that the predetermined variables provide additional information. MSE-F tests the null hypothesis that the benchmark model has an MSE less than or equal to the augmented model against the alternative hypothesis that the augmented model has smaller MSE. We use observations over the period 1963:Q4 to 1976:Q4 for the initial in-sample estimation and then generate forecast recursively for stock returns over the period 1977:Q1 to 2002:Q4. The Asy. CV column reports the asymptotic 95% critical values provided by Clark and McCracken (2001) and McCracken (1999). The BS. CV column reports the empirical 95% critical values obtained from bootstrapping, as in Lettau and Ludvigson (2001b). In particular, we first estimate a VAR (1) process of excess stock market returns and its forecasting variables with the restrictions under the null hypothesis. We then feed the saved residuals with replacements to the estimated VAR system, of which we set the initial values to their unconditional means. The ENC-NEW and MSE-F statistics are calculated using the simulated data and the whole process is repeated 10,000 times.

	Models	MSE_A / MSE_B	ENC-NEW			MSE-F		
			Statistic	Asy. CV	BS. CV	Statistic	Asy. CV	BS. CV
1	C+ $v^2_{M,t}$ + $v^2_{H,t}$ vs. C	0.96	11.86	2.09	2.98	4.46	1.52	1.28

Table 5 Forecasting Value Premium

Note: We report the OLS regression results of forecasting the one-quarter-ahead value premium using some predetermined variables over the period 1963:Q4 to 2002:Q4. The heteroskadesticity-corrected standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. $v_{M,t}^2$ is realized stock market variance; $v_{H,t}^2$ is realized variance of the value premium; $v_{MH,t}$ is realized covariance between the stock market return and the value premium; $RREL_t$ is the stochastically detrended risk-free rate; CAY_t is the consumption-wealth ratio proposed by Lettau and Ludvigson (2001b); and IV_t is a measure of idiosyncratic volatility used in Guo and Savickas (2005).

	$v_{M,t}^2$	$v_{MH,t}$	$v_{H,t}^2$	$RREL_t$	CAY_t	IV_t	$\bar{R}^2$
1		10.574	17.232*	3.178**			0.056
		(7.506)	(9.782)	(1.412)			
2		10.679	16.120		-0.301		0.033
		(7.554)	(9.891)		(0.408)		
3		10.925	15.205			0.423	0031
		(7.511)	(10.565)			(1.015)	
4	-3.719**		10.792**				0.048
	(1.627)		(4.842)				
5	-3.375**		10.383**	2.772*			0.062
	(1.633)		(4.718)	(1.420)			
6	-3.875**		9.644**		-0.398		0.048
	(1.669)		(4.711)		(0.412)		
7	-4.852**		5.081			1.414	0.057
	(1.945)		(6.273)			(1.185)	

Table 6 Realized Variance of Alternatively Measured Value Premium

Note: We report the OLS regression results of forecasting one-quarter-ahead returns using some predetermined variables over the period 1963:Q4 to 2002:Q4. The heteroskadesticity-corrected standard errors are in parentheses and ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. $v_{M,t}^2$ is realized stock market variance; $v_{H,t}^2$ is realized variance of the value premium; $v_{HB,t}^2$ is realized variance of the value premium based on big stocks; and $v_{HS,t}^2$ is realized variance of the value premium based on small stocks.

	$v_{M,t}^2$	$v_{H,t}^2$	$v_{HS,t}^2$	$v_{HB,t}^2$	$\overline{R}^2$
Panel A: Stock Market Returns					
1	8.168***	-17.038***			0.078
	(2.194)	(5.203)			
2	7.826***		-13.760***		0.061
	(2.124)		(4.795)		
3	8.174***			-17.065***	0.078
	(2.191)			(5.146)	
Panel B: The Value Premium					
4	-3.719***	10.792***			0.048
	(1.627)	(4.842)			
5	-4.224**		10.735***		0.057
	(1.654)		(3.608)		
6	-3.728**			10.825**	0.049
	(1.625)			(4.819)	

Table 7 Summary Statistics of Monthly Return Data

Note: The table reports summary statistics of the excess stock market return, R_t, and the value premium, HML_t, in percentage. Panel B reports the unconditional variance-covariance matrix in the upper triangle and the correlation coefficient in the lower triangle. Panel C reports the conditional variances and covariance, which are based on estimation of the ABEKK model in row 4, Table 9. $\sigma^2_{M,t}$ is stock market variance, $\sigma^2_{H,t}$ is variance of the value premium, and $\sigma_{MH,t}$ is covariance between the stock market return and the value premium. The sample spans the period January 1963 to February 2004. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Panel A. Summary Statistics

	Mean	Standard Deviation	Skewness	Kurtosis	Ljung-Box statistics		
					Q1	Q6	Q12
R_t	0.496	4.480	–0.485	4.930	1.226	5.620	8.790
HML_t	0.358	3.195	–0.571	9.582	10.711***	23.362***	26.739***

Panel B. Unconditional Covariance Matrix

	R_t	HML_t
R_t	20.029	–4.593
HML_t	–0.322	10.187

Panel C. Mean of Conditional Variances and Covariance

$\sigma^2_{M,t}$	$\sigma_{MH,t}$	$\sigma^2_{H,t}$
20.725	–4.919	9.595

Table 8 Specification Tests

Note: The table reports the specification tests of the various GARCH specifications described in equations (15) through (18). The sample spans the period January 1963 to February 2004.

Null hypothesis	DF	LR	p-value
Panel A. Pooling Univariate GARCH Model vs. ADC Model			
H_0: No Interaction Term	10	112.93	0.00
Panel B. ABEKK model vs. ADC Model			
H_0: $\rho_{MH} = 0$ and $\phi_{MH} = 1$	2	1.30	0.52
Panel C. BEKK Model vs. ABEKK Model			
H_0: $g_{MM} = g_{MH} = g_{HM} = g_{HH} = 0$	4	32.52	0.00
Panel D. Constant Equity Premium and Value Premium in ABEKK Model			
H_0: $\gamma_{MM} = \gamma_{MH} = \gamma_{HM} = \gamma_{HH} = 0$	4	18.49	0.00
Panel E. No Constant Terms in ABEKK Model			
H_0: $\alpha_{ER} = \alpha_{HML} = 0$	2	2.16	0.34
Panel F. Equal Risk Prices Across Assets in ABEKK Model			
H_0: $\gamma_{MM} = \gamma_{MH}$, $\gamma_{HM} = \gamma_{HH}$ $\alpha_{ER} = \alpha_{HML} = 0$	4	4.12	0.39

Table 9 Merton's ICAPM: Bivariate GARCH Model

Note: The table reports the estimation results of the Merton's ICAPM using various bivariate GARCH models described in equations (15) through (18). Unless otherwise indicated, we use the QML method and the monthly sample spanning the period January 1963 to February 2004. We use the sample period July 1926 to December 1962 in row 5 and the sample period July 1926 to February 2004 in row 6. The specifications in rows 9 and 10 are the same as those in row 4 except that we assume a *t* distribution in row 9 and a normal distribution in row 10. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. We report the log likelihood in the column under LL.

	Stock Market Returns			Value Premium			LL
	α_R	γ_{MM}	γ_{HM}	α_H	γ_{MH}	γ_{HH}	
Panel A: Pooling Univariate GARCH							
1	0.003	1.21		0.000		3.16	-2599.52
	(0.005)	(2.83)		(0.002)		(2.22)	
Panel B: ABEKK Model							
2	0.001	4.70	9.15	0.001	2.41	2.64	-2544.35
	(0.005)	(3.83)	(8.49)	(0.002)	(4.89)	(3.54)	
3		5.90***	10.3		1.89	4.63	-2545.43
		(2.01)	(7.58)		(5.40)	(3.28)	
4		4.64***	5.86***		4.64***	5.86***	-2546.41
		(1.12)	(1.77)		(1.12)	(1.77)	
5		2.46***	0.006		2.46***	0.006	-2380.88
		(0.825)	(1.66)		(0.825)	(1.66)	
6		2.47***	1.39		2.47***	1.39	-4985.31
		(0.579)	(0.894)		(0.579)	(0.894)	
Panel C: ADC Model							
7	0.002	3.33	6.76	-0.000	3.29	1.39	-2543.05
	(0.007)	(4.01)	(4.37)	(0.002)	(3.12)	(0.894)	
8		4.19***	6.05***		4.19***	6.05***	-2556.50
		(1.18)	(1.81)		(1.18)	(1.81)	
Panel D: ABEKK Model Using MLE Method							
9		4.79***	6.26***		4.79***	6.26***	–2528.70
		(1.06)	(1.66)		(1.06)	(1.66)	
10		4.64***	5.86***		4.64***	5.86***	-2546.41
		(1.11)	(1.85)		(1.11)	(1.85)	

Table 10 Parameter Estimates of the Benchmark ABEKK Model

Note: The table reports the estimation results of the ABEKK specification of equations (15) through (18) by imposing the restrictions ρ_{MH} = 0 and ϕ_{MH} =1, the same specifications as those reported in row 4 of Table 9. We also impose the ICAPM restrictions $\gamma_{MM} = \gamma_{MH}$, $\gamma_{HM} = \gamma_{HH}$, and $\alpha_R = \alpha_H = 0$. The sample spans the period January 1963 to February 2004. ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively.

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
Panel A. Mean equation of Stock Return			Panel B. Mean equation of value Premium		
γ_{MM}	4.64***	1.12	γ_{MH}	4.64***	1.12
γ_{HM}	5.86***	1.77	γ_{HH}	5.86***	1.77
Panel C. Variance equation of stock Return			Panel D. Variance equation of value Premium		
ω_{MM}	1.030***	0.240	ω_{HH}	0.658***	0.149
b_{MM}	0.933***	0.022	b_{HH}	0.871***	0.030
a_{MM}	0.250***	0.068	a_{HH}	0.244***	0.062
g_{MM}	0.125*	0.075	g_{HH}	0.340***	0.065
Panel E. Covariance equation of Stock Return and Value Premium					
ω_{MH}	-0.524**	0.244	b_{MH}	0.025	0.014
a_{MH}	-0.128**	0.065	a_{HM}	-0.103**	0.0390
g_{MH}	-0.027	0.081	g_{HM}	0.070	0.042

Table 11 Specification Tests

Note: The Table reports the standardized residuals and their second moments. $\hat{\varepsilon}_M$ is the residual of the stock market return and $\hat{\varepsilon}_H$ is for the value premium. The calculation is based on the estimation of Table 10. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

	$\hat{\varepsilon}_M$	$\hat{\varepsilon}_H$	$\hat{\varepsilon}_M \hat{\varepsilon}_M$	$\hat{\varepsilon}_M \hat{\varepsilon}_H$	$\hat{\varepsilon}_H \hat{\varepsilon}_H$
Panel A. Sample statistics					
Mean	–0.037	0.030	0.999	–0.307	0.992
Standard Deviation	1.000	0.997	2.110	1.246	1.556
Skewness	–0.692	0.050	8.107	–2.046	2.857
Kurtosis	5.354	3.447	101.388	21.208	13.499
t-statistic for mean = 0	–0.822	0.670			
t-statistic for mean = 1			–0.009	–23.250***	–0.111
Panel B. Ljung-Box statistics					
Q_1	0.920	18.222***	0.586	0.263	0.848
Q_6	5.412	22.923***	1.517	7.815	2.751
Q_{12}	7.788	28.399***	5.458	20.962	6.307

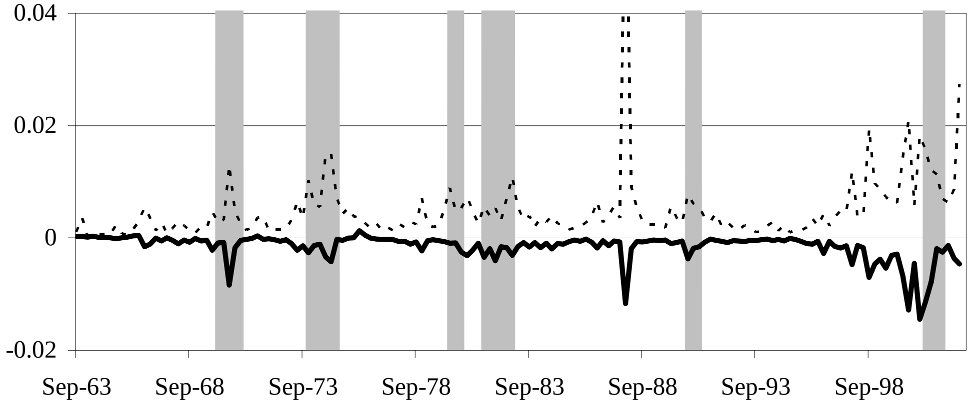

Figure 1
Realized Stock Market Variance (Dashed line) and Covariance between Stock Market Return and the Value Premium (Solid Line)

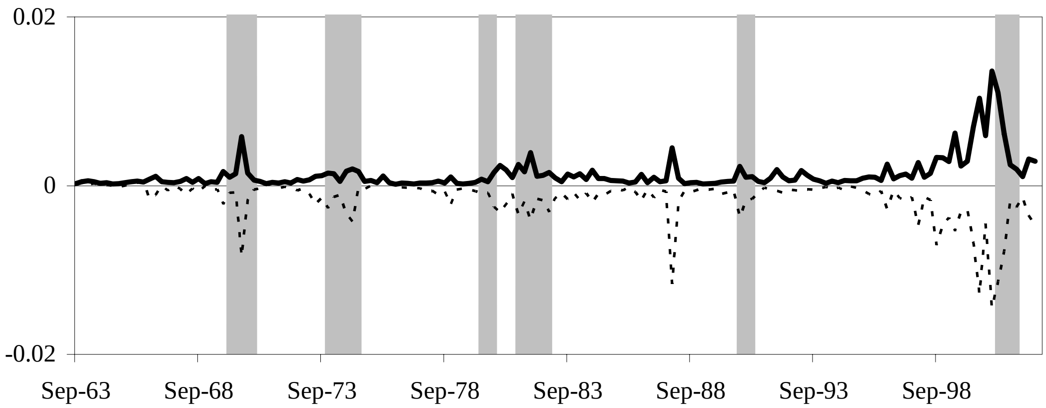

Figure 2
Realized Variance of Value Premium (Solid Line) and Covariance between Stock Market and Value Premium (Dashed Line)

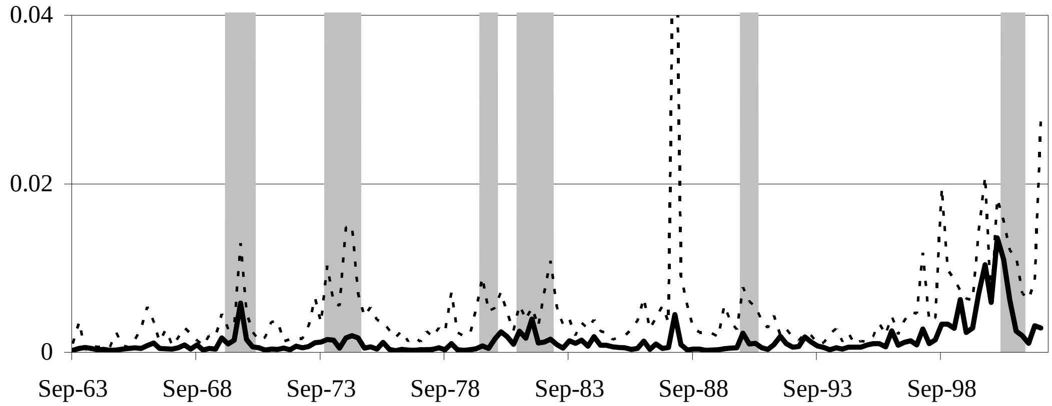

Figure 3
Realized Variance of Stock Market Return (Dashed Line) and Value Premium (Solid Line)

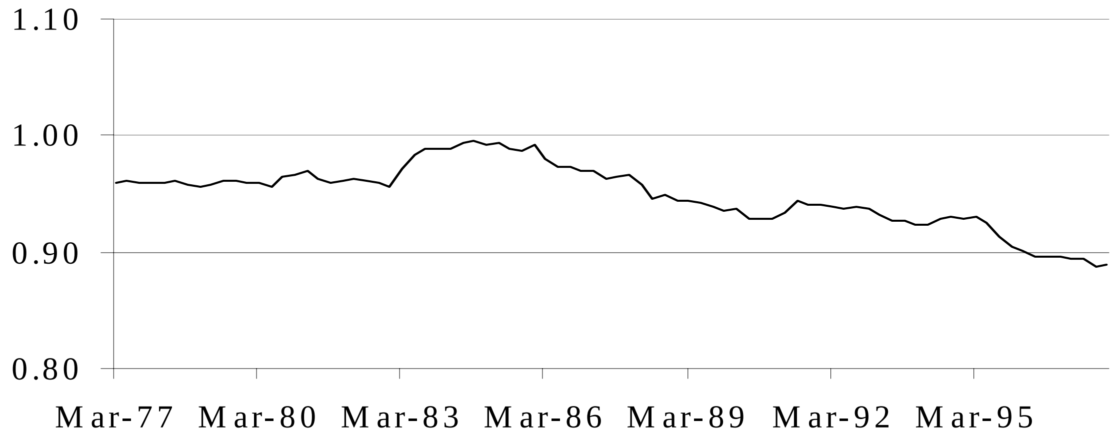

Figure 4
Recursive MSE Ratio of Augmented Model to Benchmark Model (Table 4)

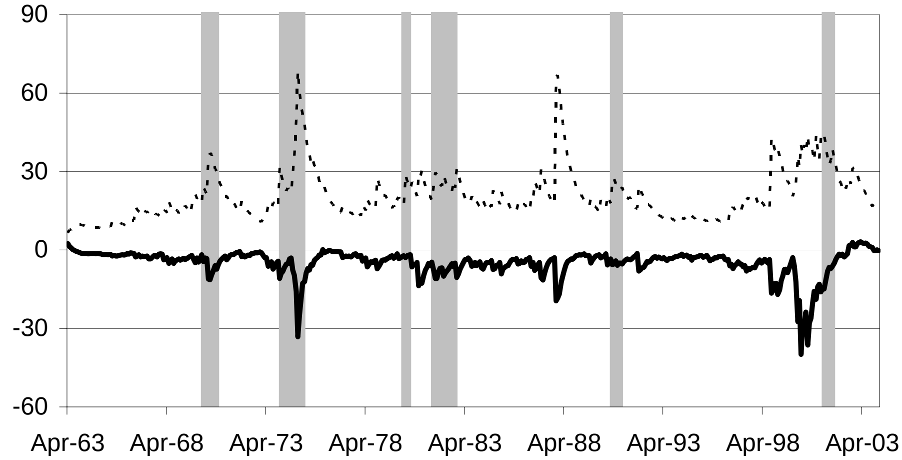

Figure 5
Conditional Stock Market Variance (Dashed Line) and Covariance between Stock Market Return and Value Premium in Estimation of Row 4, Table 9

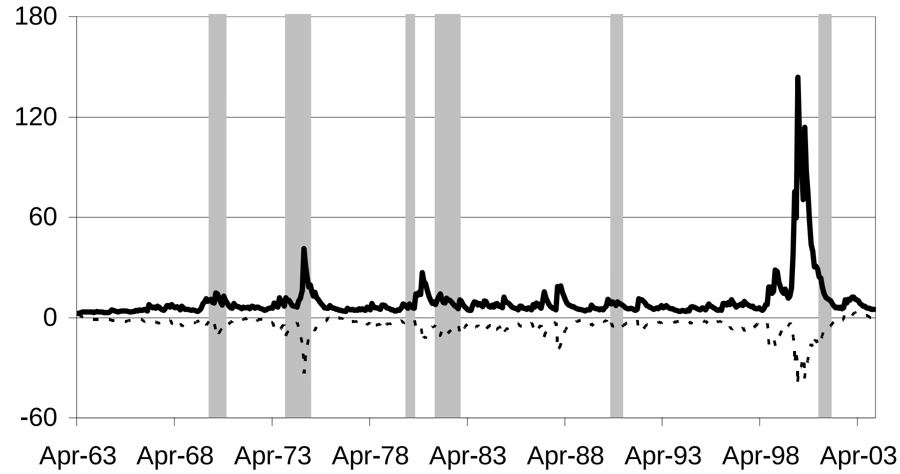

Figure 6
Conditional Variance of Value Premium (Solid Line) and Covariance between Stock Market Return and Value Premium (Dashed Line) in Estimation of Row 4, Table 9

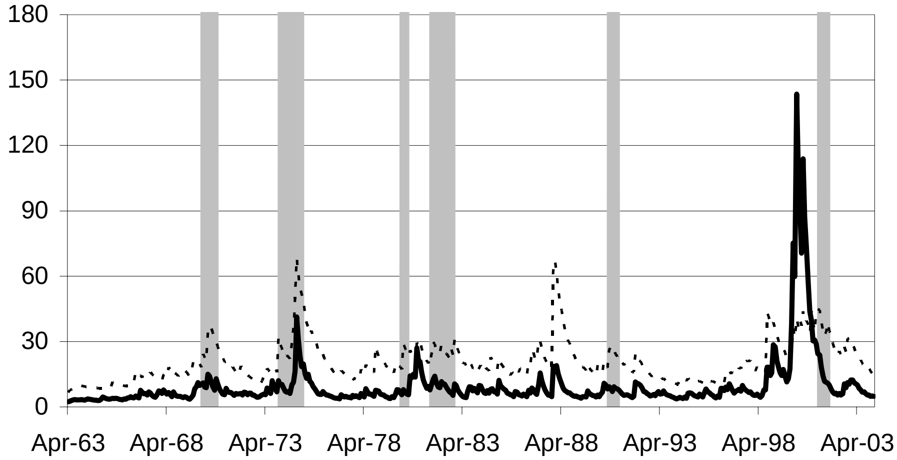

Figure 7
Conditional Variance of Stock Market Return (Dashed Line) and Value Premium (Solid Line) in Estimation of Row 4, Table 9

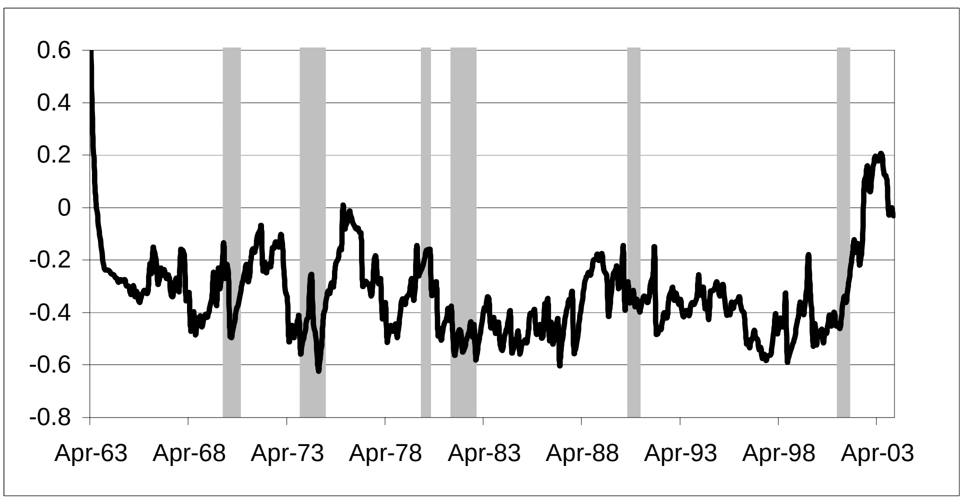

Figure 8
Coefficient of Correlation between Stock Market Return and Value Premium in Estimation of Row 4, Table 9