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The Low-Volatility Anomaly: Market Evidence on Systematic Risk vs. Mispricing

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The authors explored whether the well-publicized anomalous returns associated with low-volatility stocks can be attributed to market mispricing or to compensation for higher systematic factor risk. The results of their study, covering a 46-year period, indicate that the relatively high returns of low-volatility portfolios cannot be viewed solely as compensation for systematic factor risk. The results from their cross-sectional analyses indicate that average returns to low-volatility portfolios are determined by common variations associated with the idiosyncratic-volatility characteristic rather than factor loadings. This finding suggests that the excess returns are more likely driven by market mispricing connected with volatility as a stock characteristic.

In what is sometimes referred to as the *low-volatility anomaly*, researchers have discovered a provocative long-term connection between future stock returns and various measures of prior stock price variability, including total return volatility, idiosyncratic volatility, and beta. More to the point, researchers have documented that in both US and international markets, future stock returns of previously low-return-variability portfolios significantly outperform those of previously high-return-variability portfolios (see, e.g., Ang, Hodrick, Xing, and Zhang 2006, 2009; Clarke, de Silva, and Thorley 2006; Blitz and van Vliet 2007; Baker, Bradley, and Wurgler 2011; Li, Sullivan, and Garcia-Feijóo 2014). These empirical findings are particularly intriguing because economic theory dictates that higher expected return compensates for higher risk. Thus, these findings highlight the need to gain a better understanding of the underpinnings of this curious anomaly. An explanation for its existence, however, remains elusive, especially regarding whether it is driven by systematic risks or investor mispricing. In our study, we sought to gain fruitful insight into the low-volatility anomaly—which, as we show later in the article, comes predominantly from the underperformance of the highest-volatility stocks—by examining whether it can be attributed largely to market mispricing or to compensation for higher systematic (undiversifiable) risk.

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The Low-Volatility Anomaly

Focusing on market beta, Black (1972) offered an early theoretical interpretation of why low-risk stocks might do so well relative to high-risk stocks. He showed that a delegated agent mispricing arising from such borrowing restrictions as margin requirements might cause low-beta stocks to outperform. More recently, some have argued that the low-volatility anomaly is likely due to some pervasive systematic risk factor(s) directly associated with volatility. For example, Clarke, de Silva, and Thorley (2010) suggested that idiosyncratic volatility (and total volatility) is a potential additional risk factor to which portfolio managers should pay attention. They found that the excess return to idiosyncratic-volatility stocks is immaterial over the full sample period (1931–2008), implying that investors have historically not been rewarded for bearing such risk over the long haul. For more recent years (1983–2008), however, they found that exposure to low-idiosyncratic-volatility stocks benefited investors, although the evidence of cross-sectional idiosyncratic volatility is weak.

Ang et al. (2009) found evidence of an idiosyncratic-volatility anomaly in numerous countries and discovered that the effect is highly correlated with that in the United States. Arguing that such an effect could be driven by latent systematic risks, they showed that abnormal returns generated by idiosyncratic-volatility-based portfolio strategies in international markets strongly co-move with those in US markets, implying a common risk factor. They stated that "the large commonality in co-movement…suggests that broad, not easily diversifiable factors lie behind this effect" (p. 2). This finding of co-movement suggests that the return-predictive power of idiosyncratic risk is likely due to some pervasive risk factor.

Still others have argued that the low-volatility anomaly is likely due to mispricing, perhaps associated with an imperfection such as investor irrationality connected with idiosyncratic volatility. In the case of mispricing, the profit opportunity may be ephemeral as investors come to understand their cognitive error. Or it could be a more lasting mispricing, supported over time by the high costs of arbitraging away the anomalous returns. For instance, Li et al. (2014) showed that the efficacy of trading the low-volatility factor is somewhat limited owing to the high costs to arbitrage (e.g., high transaction costs) that are directly associated with attempting to extract the anomalous excess returns.

Perhaps the anomalous effect is supported by behavioral considerations. Similar to Black (1972), Baker et al. (2011) proposed an explanation consistent with biases that originate in investor behavior based on a delegated asset management model. They showed that institutional client mandates discourage arbitrage activity that would otherwise potentially eliminate the low-volatility effect. Frazzini and Pedersen (2014) showed that a betting against beta (BAB) factor, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns. They found that more constrained investors hold riskier assets, leading them to bid up high-beta assets. They found empirically that high beta is associated with low alpha for US equities, 20 international equity markets, Treasury bonds, corporate bonds, and futures.

Merton (1987) offered an interesting explanation for why investors would demand higher returns for taking on higher idiosyncratic risk, suggesting that idiosyncratic risk is positively related to expected return when investors cannot fully diversify their portfolio; that is, investors demand higher compensation from stocks with higher idiosyncratic volatility to compensate for imperfect diversification. Interestingly, the empirical evidence in Ang et al. (2009) and Clarke et al. (2010) runs counter to Merton's (1987) prediction.[1] Collectively, these findings highlight the importance of a formal investigation into the underlying economic question of whether the various low-risk effects are associated with market mispricing or systematic risk.

In our study, instead of debating whether previously low-volatility stocks can explain future returns empirically, we asked whether there is a pervasive systematic factor directly associated with return variability. We thus aimed to shed light on the outperformance of securities with low idiosyncratic volatility, a phenomenon reported in Ang et al. (2006, 2009). Put differently, the abnormal returns that researchers have documented for low-volatility portfolios could be due to the portfolios' exposure to some not-yet-understood common risk component. For instance, high-volatility stocks may offer consumption-hedging benefits by performing better during weak economic conditions. Theoretically, investors would be willing to pay more for stocks with such hedging benefits. In contrast, investors would buy only low-volatility stocks if they offered a higher expected return, given that their (not-yet-well-understood) exposure to systematic risk causes them to deliver poor returns when cash flows are most valued by investors (e.g., during recessions). Alternatively, investors may prefer high-volatility stocks to low-volatility stocks, perhaps owing to cognitive biases or some other not-yet-understood reason.[2]

To determine which of the two explanations (mispricing or systematic risk) better elucidates the low-risk effect, we investigated whether the low-risk anomaly represents returns to some not-yet-identified risk factor or instead is related to the characteristic of low risk itself (e.g., Cohen and Polk 1995; Daniel and Titman 1997, 1998; Davis, Fama, and French 2000; Daniel, Titman, and Wei 2001; Grundy and Martin 2001). These researchers applied specific test methods to identify the source of such well-known anomalies as size, book-to-market, and momentum. We relied on the same methodologies in examining the low-volatility anomaly to test whether the previously identified differential returns of high- and low-volatility stocks can be attributed to factor loadings and/or to company characteristics. In other words, we sought to determine empirically whether the low-volatility anomaly is associated with a mispricing or a pervasive systematic risk. In the language of Daniel and Titman (1997, 1998), we performed tests regarding characteristics versus covariances.[3] Using such tests, we were able to examine whether variations in the loadings on a factor created on the basis of volatility (Fama and French 1993) can explain future stock returns after controlling for actual return variability.

For the systematic risk explanation of the low-volatility anomaly to be valid, stocks with a high loading on the low-volatility factor should outperform stocks with a low loading on the low-volatility factor. This pattern should be observed irrespective of the absolute level of stock volatility. If, however, after controlling for the observed level of return variability, loadings on the low-volatility factor are unable to explain cross-sectional stock returns, we can reasonably conclude that the low-volatility anomaly is consistent with market mispricing.

In our study, however, we did not intend to empirically identify the source of any possible latent systematic risk or explanation for any market mispricing. One attraction of the asset-pricing methodologies of Daniel and Titman (1997, 1998) is that they allow researchers to be agnostic about the source of the anomalous effect. For example, if an anomaly is truly due to systematic risk, this approach can still capture a latent systematic risk and attribute it to the anomaly, even if the source of the risk is unknown (i.e., not among those already identified in the literature).

Our results indicate that (1) the low-volatility anomaly is not due to some systematic risk factor and (2) there is no return premium associated with a factor formed on the basis of volatility. These findings suggest that the abnormal returns identified in the literature cannot be viewed as compensation for systematic risk. Put differently, we found that the pricing of the characteristic itself can better explain the outperformance of low-volatility stocks, suggesting a market mispricing.

Our findings provide insights into the well-documented excess returns to various low-risk anomalies—insights that can enable investors to improve portfolio construction and risk management via a better understanding of the source of the anomalous returns over time and across companies. In our study, we drew heavily on the rigorous methods in the asset-pricing literature to shed light on whether the return-predictive power of idiosyncratic risk derives from systematic risk or from mispricing.

Data and Sample

We obtained stock return data from the Center for Research in Security Prices (CRSP) for all stocks trading on the NYSE, AMEX, and NASDAQ over 1963–2011. For delisted companies, the CRSP monthly return file does not include the returns from the delisting month unless the delisting date is at month-end. We fetched the returns in the delisting month and the market capitalization on the delisting date from the CRSP daily return file and combined those returns with the delisting returns to create the effective delisting month returns. If, however, the delisting was for performance-related reasons, we set the delisting return equal to –55% if trading on NASDAQ or –30% if trading on the NYSE or AMEX (for an analysis of CRSP delisting bias, see Shumway 1997; Shumway and Warther 1999).

We followed the most recent literature by focusing on idiosyncratic volatility, which studies have shown is negatively associated with subsequent stock returns. We measured idiosyncratic volatility (IVOL) each month as the standard deviation of the residual returns from the Fama–French three-factor model by regressing the daily returns of individual stocks in excess of the one-month T-bill rate ($R_{i,t} - R_{f,t}$) on the returns to the common factors related to size and book-to-market. In other words, we performed the following time-series regression for each stock i:

$$R_{i,t} - R_{f,t} = a_i + b_i(R_{M,t} - R_{f,t}) + s_i\text{SMB}_t + h_i\text{HML}_t + \varepsilon_{i,t},$$

where $R_{M,t} - R_{f,t}$, SMB, and HML represent the Fama–French market, size, and value factors, respectively. We required a minimum of 15 observations for model estimation. With this requirement, we omitted the most illiquid stocks from our results, thus minimizing the likelihood that our results are biased toward stocks that trade infrequently. We correlated the idiosyncratic risk from the current month with the subsequent monthly returns (inclusive of dividends).

Following Fama and French (1993) and Daniel and Titman (1997), we constructed the IVOL-based factor as a zero-investment factor-mimicking portfolio. At the end of each month, we sorted stocks into size (market cap) terciles using NYSE breakpoints; we sorted stocks into terciles on the basis of the IVOL characteristic. We obtained value-weighted monthly returns on nine portfolios: three size portfolios for each of the three portfolios based on the IVOL characteristic. We then equally weighted each IVOL portfolio across the size terciles to obtain returns on three size-independent IVOL portfolios. To calculate the returns on the zero-cost portfolio representing the IVOL-based factor, we subtracted the monthly return on the high-IVOL portfolio from the monthly return on the low-IVOL portfolio.

To estimate factor loadings (betas) on the IVOL factor, we followed the approach used by Daniel and Titman (1997, 1998). We conducted rolling regressions of monthly excess stock returns on the three Fama–French (1993) factors plus the IVOL factor over the previous 36 months (24 months minimum). The portfolio weights of the factor portfolios were constant (based on factor weights each month); that is, to calculate the returns of constant-weight factor portfolios, we applied the portfolio weights of the factors for each month t to returns from date $t - 37$ to $t - 1$. Our estimated loadings on the IVOL factor are pre-formation IVOL betas (β_{IVOL}). If covariances are stationary over time, factor loadings estimated in this way should be good predictors of future betas on the IVOL factor (later in the article, we present evidence confirming that is indeed the case). We obtained IVOL factor betas for January 1966–December 2011 (552 months).

Our approach allowed us to separate low-IVOL stocks with high and low loadings on the IVOL factor. If the risk-based explanation for the higher observed returns of low-IVOL stocks is correct, a low-IVOL stock with a low-IVOL factor loading should have a low average return. In contrast, if characteristics rather than factor loadings determine prices, a low-IVOL stock should have a high return regardless of its loading.

Tests and Results

Table 1 reports summary statistics for the relevant variables, including a correlation matrix. Panel B of Table 1 shows that the absolute correlations between the IVOL factor and the other portfolio characteristics are moderately high to high. Note that, as expected, the IVOL factor is negatively contemporaneously related to the market return (–0.56), indicating that when low-IVOL stocks outperform high-IVOL stocks (i.e., the IVOL factor is positive), market returns are relatively low; conversely, high-IVOL stocks outperform when market returns are relatively high.

As part of our analysis, we later formed quintile portfolios on the basis of IVOL factor betas (i.e., exposure to IVOL risk). The correlation between the IVOL factor and the difference between the returns on the high- and low-β_{IVOL} quintiles of stocks is 0.68; that is, stocks with a high exposure to a possible source of systematic risk outperform when the risk factor premium is relatively high. The correlation between the IVOL factor and the difference between the high and low quintiles based on market beta (β_{CAPM}) is –0.79. In addition, the difference between the high and low quintiles based on β_{IVOL} has a correlation with the difference between the high- and low-β_{CAPM} quintiles of –0.71.[4] The negative sign likely occurs by construction (the IVOL factor is low minus high in order to measure a risk premium).

The relatively high absolute correlations in Table 1 are not surprising. Measures of stock return variability are likely to be correlated, and the summary statistics reported in Table 1 do not control for important company characteristics, such as market capitalization. We conducted a variety of analyses designed to disentangle the effects of IVOL risk from other well-known factors.

Figure 1 plots the cumulative stock returns on both the market (from Kenneth French's website[5]) and the zero-cost IVOL factor, as described earlier. As Figure 1 shows, the IVOL factor has outperformed the market portfolio during market declines and has tended to underperform during rising markets, especially in the current decade. This finding is consistent with Garcia-Feijóo, Kochard, Sullivan, and Wang (2015), who found that a zero-cost IVOL factor portfolio has a negative market beta, meaning that the portfolio is short the market over time on average.

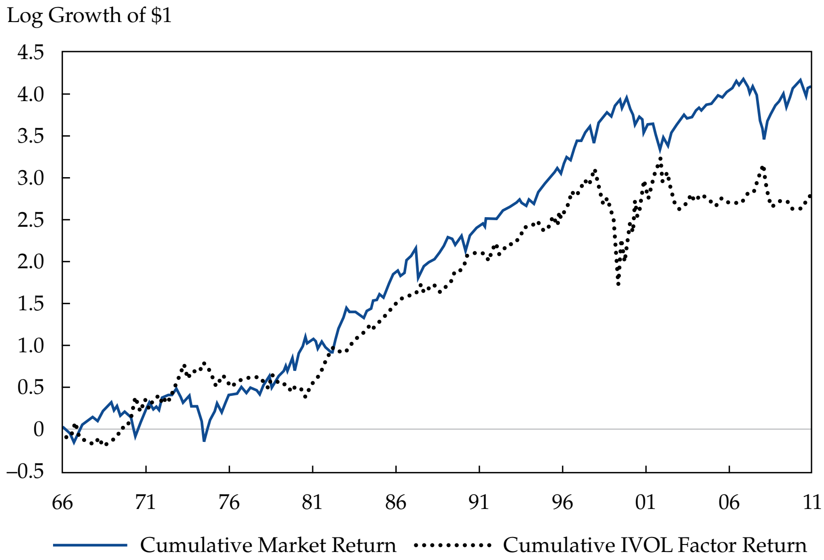
Table 2 provides further descriptive information for our key variables sorted into quintiles on the basis of the IVOL characteristic; the sample period is 1966–2011. "EWRet" and "VWRet" represent the average raw returns for equal-weighted and value-weighted quintiles, respectively. We computed "IVOL" as the IVOL from regressions of excess returns on the three Fama–French factors (using daily observations for a minimum of 15 days), multiplying it by the square root of the number of trading days in a month to convert it to a monthly measure. As Table 2 shows, average

Table 1. Summary Statistics for Relevant Variables, 1966–2011

Variable	IVOL Factor	High β_{IVOL} – Low β_{IVOL}	High β_{CAPM} – Low β_{CAPM}	$R_M - R_f$	HML	SMB
A. Statistic						
Mean	0.66%	–0.01	0.02	0.41	0.37	0.25
Standard deviation	5.53	4.43	6.50	4.64	2.98	3.21
Quartile 3	3.02	2.51	3.72	3.56	1.78	2.17
Median	0.63	0.02	–0.26	0.75	0.37	0.07
Quartile 1	–1.62	–2.29	–3.61	–2.31	–1.30	–1.59
B. Correlations						
IVOL factor	1.00	0.68***	–0.79***	–0.56***	0.51***	–0.61***
High β_{IVOL} – Low β_{IVOL}		1.00	–0.71***	–0.47***	0.32***	–0.42***
High β_{CAPM} – Low β_{CAPM}			1.00	0.67***	–0.50***	0.51***
$R_M - R_f$				1.00	–0.31***	0.31***
HML					1.00	–0.23***

***Significant at the 1% level.

Figure 1. Cumulative Monthly Returns on Both the Market and the IVOL Factor



value-weighted returns (VWRet) decline from the lowest-IVOL quintile to the highest-IVOL quintile, a finding consistent with the notion that low-risk stocks outperform high-risk stocks, on average. The so-called low-volatility effect is especially prominent in the highest-IVOL quintile, with the IVOL portfolios having significantly lower value-weighted returns than the other, lower-IVOL quintiles, which have similar returns. This finding suggests that the effect may be due to some perhaps constrained (Frazzini and Pedersen 2014) investors bidding up the price of high-volatility stocks. The lowest-IVOL quintile shows a VWRet of 0.88% a month, and the highest-IVOL quintile shows a VWRet of 0.24% a month. Column 2 of Table 2, however, shows that on an equal-weighted market return (EWRet) basis, the performance of the lowest-IVOL quintile stocks (1.14% a month) is lower than that of the other quintiles, rising to 1.78% for the highest-IVOL quintile. These findings are consistent with prior research (see, e.g., Li et al. 2014).

Not surprisingly, monthly idiosyncratic volatility (the ranking variable) increases from the lowest to the highest quintile. Average market betas also increase, which indicates that both measures of risk are positively associated. The last column of Table 2 reports evidence of a negative unconditional association between IVOL (the characteristic) and β_{IVOL}: The average β_{IVOL} is 0.15 for the lowest quintile and –0.31 for the highest quintile. This finding is important because in order to be able to distinguish between risk and mispricing as possible explanations for the low-IVOL anomaly, there needs to be dispersion in β_{IVOL} that is unrelated to IVOL as a characteristic. Thus, we probed more deeply into the potential underpinnings of the low-risk anomaly.

Cross-Sectional Regressions. We began our formal investigation by applying an extension of the monthly Fama–MacBeth (1973) cross-sectional regressions in which we regressed individual stock returns

Table 2. Risk and Return Characteristics of IVOL Portfolios, 1966–2011 (*t*-statistics in parentheses)

IVOL Quintile	EWRet	VWRet	IVOL	β_{CAPM}	β_{IVOL}
1 (Low)	1.14%	0.88%	4.37%	0.79	0.15
2	1.31	0.92	7.46	1.02	0.06
3	1.41	0.94	10.50	1.20	–0.05
4	1.44	0.83	14.86	1.34	–0.19
5 (High)	1.78	0.24	28.22	1.40	–0.31
High – Low	0.64**	–0.64**	23.84***	0.61***	–0.46***
	(2.11)	(–2.20)	(49.20)	(29.44)	(–34.45)

Note: Reported averages are computed as time-series averages of cross-sectional means.

**Significant at the 5% level.

***Significant at the 1% level.

on the loadings on the IVOL factor (β_{IVOL}) and the level of the IVOL characteristic while controlling for the well-known size and style effects. We measured size (ME) as the logarithm of the equity market capitalization at the end of the prior month, and we measured book-to-market (BEME) as the logarithm of 1 plus the book-to-market ratio of equity (computed as in Fama and French 1992); we used accounting data for the prior fiscal year and market capitalization as of the end of the prior calendar year.

Table 3 presents the results. Columns 1 and 2 show that both the IVOL characteristic and the loading on the IVOL factor (β_{IVOL}) are insignificantly related to subsequent stock returns when other variables are not controlled for (t = 0.99 and t = –0.31, respectively). Columns 3 and 4 show the results when the common control variables of size and style are included, with column 3 showing that β_{IVOL} remains insignificant. In contrast, columns 4 and 5 show that the IVOL characteristic can predict subsequent stock returns at the 1% significance level with the inclusion of other control variables. Columns 1–5 report results for the full sample; columns 6–8 report results for three subperiods. In our linear regression analysis, we found evidence of a strong IVOL effect prior to 1990, which disappears in the more recent period. In all our regressions, β_{IVOL} is never significant. Thus, the results from our cross-sectional regressions indicate that average subsequent returns over the study period are determined by common variations associated with the IVOL characteristic rather than factor loadings. This analysis suggests that the return-predictive power of IVOL is best explained by a market mispricing rather than by some pervasive risk factor.

Table 4 reports the results of our rank portfolio test, which we conducted to further explore the performance of strategies based on the IVOL characteristic and the IVOL-based factor loadings. This test is commonly used to assess whether the return differences generated by the characteristic and the factor loading differ across quintiles (i.e., nonlinearly). We assigned companies equally to quintile portfolios according to the magnitude of the prior month's IVOL characteristic or β_{IVOL}. We then calculated the following month's equal-weighted return and value-weighted return for each quintile portfolio. We separately measured the abnormal returns on the quintile spread portfolio—that is, the difference portfolio between the highest- and lowest-ranked quintiles. We calculated the abnormal returns for each portfolio using the intercept from the Fama–French (1993) three-factor model whose dependent variables are the monthly returns of these portfolios in excess of the risk-free rate.[6]

Table 4 shows that sorting solely on β_{IVOL} generates insignificant abnormal returns for the equal-weighted and value-weighted quintile portfolios (e.g., the zero-cost spread, or difference, portfolios have

Table 3. Monthly Fama–MacBeth Regressions of Stock Returns on Both IVOL and β_{IVOL}, 1966–2011 (t-statistics in parentheses)

	Coefficient							
	1	2	3	4	5	6	7	8
Sample period	1966–2011	1966–2011	1966–2011	1966–2011	1966–2011	1966–1989	1990–2011	1990–2006
Variable								
IVOL characteristic	1.03%			–1.79%***	–1.76%***	–3.83%***	0.60%	1.15%
	(0.99)			(–2.87)	(–2.90)	(–4.66)	(0.79)	(1.29)
β_{IVOL}		–0.02%	0.03%		0.02	0.02	0.02	0.06
		(–0.31)	(0.91)		(0.64)	(0.43)	(0.49)	(1.40)
Log(ME)			–0.17***	–0.18***	–0.18***	–0.17***	–0.19***	–0.21***
			(–3.81)	(–4.80)	(–4.85)	(–2.99)	(–4.10)	(–3.87)
Log(BEME)			0.35**	0.34**	0.33**	0.37***	0.28	0.36
			(2.41)	(2.44)	(2.36)	(2.63)	(1.12)	(1.26)
β_{CAPM}			0.08	0.09	0.11	–0.01	0.24	0.14
			(0.73)	(0.89)	(1.08)	(–0.11)	(1.53)	(0.87)
Intercept	1.13***		1.86***	2.02***	2.02***	2.07***	1.97***	2.20***
	(5.21)		(4.73)	(6.28)	(6.22)	(4.47)	(4.35)	(4.57)

Notes: Reported coefficient estimates are time-series means of estimated parameters from monthly cross-sectional regressions (in percentages). ME is equal to prior-month market capitalization (price times number of shares outstanding); BEME is equal to book equity at the prior fiscal year-end (computed as in Fama and French 1992) divided by market capitalization at the end of the prior calendar year. There is an average of 2,600 observations in each cross section (a minimum of 1,055, and a maximum of 3,712).

**Significant at the 5% level.

***Significant at the 1% level.

Table 4. Monthly Fama–French Factor-Adjusted Returns of Quintile Portfolios, 1966–2011 (*t*-statistics in parentheses)

	Ranking Variable			
	IVOL Characteristic		β_{IVOL}	
Weighting	EW	VW	EW	VW
1 (Low)	0.19%***	0.10%**	0.27%**	–0.08%
	(3.49)	(2.23)	(2.34)	(–0.72)
2	0.23***	0.02	0.20***	0.63
	(4.20)	(0.42)	(2.84)	(1.15)
3	0.25***	–0.03	0.21***	0.03
	(4.63)	(–0.43)	(3.86)	(0.64)
4	0.20**	–0.25**	0.23***	0.01
	(2.52)	(–2.56)	(4.37)	(0.24)
5 (High)	0.44***	–0.99***	0.34***	0.07
	(2.60)	(–7.06)	(5.66)	(0.87)
High – Low	0.25	–1.09***	0.06	0.15
	(1.30)	(–6.67)	(0.57)	(0.92)

Note: Table 4 reports the coefficient estimates and *t*-statistics for the intercepts ("alphas") of the Fama–French (1993) three-factor model (in percentages).

**Significant at the 5% level.
***Significant at the 1% level.

insignificant coefficient estimates). In contrast, sorting solely on the IVOL characteristic generates a significant difference in returns for the value-weighted IVOL characteristic spread portfolio, with a significant coefficient estimate of –1.09 ($t = -6.67$). The equal-weighted IVOL characteristic spread portfolio is insignificant. From the coefficient estimate of the difference portfolio, adjusted for the three Fama–French (1993) factors, we calculated the implied annualized abnormal monthly value-weighted return to a strategy that goes long low-volatility stocks and short high-volatility stocks as 13.89% [= $(1 + 1.09\%)^{12} - 1$].

Double-Sorting on Both Characteristics and Factor Loadings. We formed "characteristic-balanced" portfolios to test whether the IVOL characteristic or the IVOL factor loadings explain future stock returns—yet another approach to differentiating the market inefficiency and risk factor explanations. For each month, we sorted stocks into two groups on the basis of prior-month market capitalization (ME) using NYSE breakpoints and then into three groups on the basis of prior-month IVOL characteristic. Within each of the resulting six categories, we assigned stocks to quintiles on the basis of (pre-formation) β_{IVOL}. We computed value-weighted average returns for each quintile and for the difference between the high- and low-β_{IVOL} quintiles. As noted by Daniel and Titman (1997, 1998), in tests where factors are constructed from characteristics shown to predict returns, the factor loadings may appear to predict stock returns even though their predictive power is not due to systematic risk because the characteristic and the constructed factor tend to correlate positively. Should the IVOL factor loading explain the cross-sectional variation of stock returns, as measured by the significance of the quintile spread portfolio returns, then the predictive ability of the IVOL characteristic would likely be due to systematic risk. In contrast, the mispricing hypothesis requires that the IVOL factor loadings have no additional return-predictive power associated with the various characteristic-balanced IVOL portfolios.

Panel A of **Table 5** reports the monthly average IVOL characteristic of the stocks in each portfolio. A quick review reveals no differences across the increasing β_{IVOL} quintiles within each IVOL characteristic and size category; that is, the portfolios in each row are similar in terms of the characteristic but differ in terms of pre-formation β_{IVOL}. Panel B reports average post-formation IVOL factor loads, computed from monthly regressions of excess returns (reported in Panel C) on the three Fama–French factors and the IVOL factor. The pre-formation estimates forecast future β_{IVOL} as evidenced by the universal increase in average post-formation β_{IVOL} values as we move from Quintile 1 to Quintile 5 for each pre-formation β_{IVOL} quintile. That is, the pre-formation sorts generate noticeable differences between the *ex post* betas of the high- and low-β_{IVOL} quintiles. The important conclusion from Panels A and B is that the two extreme β_{IVOL} quintiles are different in terms of *ex post* beta exposure but not in terms of the IVOL characteristic. Although not shown, the differences are larger in terms of pre-formation betas.

Panel C of Table 5 reports value-weighted excess returns on the 30 portfolios.[7] In each row from left to right, portfolios increase in risk as measured by β_{IVOL} but do not differ in terms of characteristics (IVOL and size). If the low-volatility effect is due to systematic risk, we would expect stock returns to be significantly higher for high-β_{IVOL} portfolios. However, the insignificant results in the rightmost column, which shows the difference between the average returns of the low-β_{IVOL} quintile and those of the high-β_{IVOL} quintile, indicate that the returns are not related to exposure to the IVOL factor (after controlling for characteristics). The last row of Panel C reports the returns of a strategy that goes long high-IVOL stocks and short low-IVOL stocks of equal size. We focused on small stocks because the average monthly number of stocks in the high-IVOL big group is only 12. Low-IVOL stocks earn higher excess returns than high-IVOL stocks across beta quintiles, significantly so in Quintiles 2–4.

Panel D of Table 5 reports abnormal returns (alphas) of the regressions of value-weighted excess returns on the three Fama–French factors plus the

Table 5. Factor-Adjusted Returns Sorted on the IVOL Characteristic and IVOL-Based Factor Loadings, 1966–2011 (*t*-statistics in parentheses)

		Pre-formation β_{IVOL}					
IVOL Rank	Size Rank	1	2	3	4	5	5 – 1
A. Average of monthly IVOL							
Low	Small	5.63	5.38	5.30	5.39	5.51	
Low	Big	5.84	5.46	5.31	5.28	5.46	
Medium	Small	10.96	10.68	10.55	10.54	10.70	
Medium	Big	10.43	10.02	9.91	9.82	9.92	
High	Small	24.92	23.27	22.62	22.52	23.78	
High	Big	18.95	18.45	18.02	17.68	17.41	
Average		12.79	12.21	11.95	11.87	12.13	
B. Average of post-formation β_{IVOL}							
Low	Small	0.21	0.27	0.27	0.30	0.30	
Low	Big	0.02	0.12	0.17	0.20	0.18	
Medium	Small	–0.15	0.05	0.15	0.16	0.10	
Medium	Big	–0.41	–0.13	–0.09	0.02	0.00	
High	Small	–0.79	–0.46	–0.28	–0.25	–0.27	
High	Big	–1.18	–0.98	–0.68	–0.49	–0.54	
Average		–0.25	–0.19	–0.08	–0.01	–0.04	
C. Value-weighted average of monthly excess returns							
Low	Small	0.80%***	0.84%***	0.83%***	0.87%***	0.82%***	0.02%
		(3.47)	(4.26)	(4.43)	(4.49)	(4.11)	(0.16)
Low	Big	0.60***	0.40**	0.45**	0.38**	0.47***	–0.13
		(2.66)	(2.13)	(2.52)	(2.16)	(2.60)	(–0.90)
Medium	Small	0.90***	0.81***	0.90***	0.90***	0.85***	–0.05
		(2.76)	(2.94)	(3.53)	(3.48)	(3.19)	(–0.39)
Medium	Big	0.61*	0.58**	0.49*	0.34	0.33	–0.28
		(1.91)	(2.19)	(1.92)	(1.46)	(1.34)	(–1.38)
High	Small	0.32	0.35	0.35	0.41	0.49	0.17
		(0.75)	(0.94)	(1.08)	(1.25)	(1.45)	(0.92)
High	Big	–0.33	–0.31	–0.16	0.02	0.40	0.73*
		(–0.69)	(–0.75)	(–0.42)	(0.06)	(1.12)	(1.95)
Average		0.49	0.44	0.48	0.49	0.56	0.07
							(0.57)
High small – Low small		–0.48	–0.49**	–0.47**	–0.46**	–0.32	
		(–1.64)	(–2.02)	(–2.29)	(–2.32)	(–1.49)	
D. Alphas on three factors plus IVOL factor							
Low	Small	–0.05%	–0.02%	0.01%	0.01%	–0.04%	0.02%
		(–0.64)	(–0.31)	(0.11)	(0.14)	(–0.60)	(0.17)
Low	Big	0.16*	–0.09	–0.04	–0.12	–0.05	–0.21
		(1.77)	(–1.39)	(–0.68)	(–1.59)	(–0.55)	(–1.55)
Medium	Small	0.20**	–0.01	0.05	0.03	0.04	–0.16
		(1.96)	(–0.13)	(0.60)	(0.39)	(0.56)	(–1.40)
Medium	Big	0.41***	0.19	0.07	–0.12	–0.14	–0.55***
		(2.65)	(1.68)	(0.65)	(–1.18)	(–1.18)	(–2.67)
High	Small	0.11	–0.14	–0.23**	–0.20**	–0.07	–0.19
		(0.73)	(–1.20)	(–2.40)	(–2.13)	(–0.68)	(–1.17)
High	Big	–0.17	–0.16	–0.18	–0.19	0.32	0.49
		(–0.60)	(–0.70)	(–0.85)	(–0.81)	(1.45)	(1.30)
Average		–0.03	–0.04	–0.05	–0.10	0.01	
High small – Low small		0.17	–0.12	–0.24**	–0.21**	–0.03	
		(–0.98)	(–0.99)	(–2.23)	(–1.96)	(–0.28)	

Note: Table 5 reports value-weighted monthly excess returns (in percentages), idiosyncratic volatility, and the intercepts (in percentages) and β_{IVOL} estimates of time-series regressions of the value-weighted excess returns on the three Fama–French (1993) factors and the IVOL factor.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

IVOL factor. If the low-volatility effect is due to systematic risk, abnormal returns should be zero after adjusting for factor risk. In contrast, if the effect is due to the characteristic, mean returns would be independent of variation in the factor loadings. Thus, alphas would tend to be positive for low-β_{IVOL} portfolios because the factor model would predict lower-than-realized returns on average, and alphas would tend to be negative for high-β_{IVOL} portfolios because the factor model would predict higher-than-realized returns on average. As shown in Panel D, alphas tend to be positive for the low-β_{IVOL} portfolios and negative for the high-β_{IVOL} portfolios—though the alphas are often insignificant.

As our earlier results from Table 3 indicate, however, there is a strong low-IVOL effect in the 1966–89 subperiod, but the effect disappears after 1990. Accordingly, to better understand what underlies the effect, we focused on the 1966–89 subperiod and report the results in **Table 6**. Panel A presents the value-weighted excess returns on the 30 portfolios for the subperiod. Whereas excess returns tend to be

Table 6. Regression Results for the Characteristic-Balanced Portfolios, 1966–1989 (*t*-statistics in parentheses)

		Pre-formation β_{IVOL}					
IVOL Rank	Size Rank	1	2	3	4	5	5 – 1
A. Value-weighted average of monthly excess returns							
Low	Small	0.82%**	0.80%***	0.70%***	0.87%***	0.79%***	–0.04%
		(2.49)	(2.86)	(2.63)	(3.13)	(2.74)	(–0.33)
Low	Big	0.42	0.30	0.34	0.25	0.28	–0.13
		(1.36)	(1.09)	(1.32)	(0.99)	(1.14)	(–0.71)
Medium	Small	0.79*	0.78**	0.90**	0.89**	0.84**	0.05
		(1.79)	(2.01)	(2.46)	(2.38)	(2.20)	(0.34)
Medium	Big	0.34	0.47	0.51	0.45	0.35	0.01
		(0.85)	(1.33)	(1.55)	(1.42)	(1.06)	(0.05)
High	Small	–0.12	–0.06	0.18	0.11	0.16	0.28
		(–0.23)	(–0.13)	(0.42)	(0.25)	(0.37)	(1.31)
High	Big	–0.72	–0.61	0.01	0.21	0.19	0.91**
		(–1.41)	(–1.29)	(0.03)	(0.49)	(0.47)	(2.38)
Average		0.25	0.28	0.44	0.46	0.43	0.18
							(1.13)
Average excluding high and big		0.45	0.46	0.52	0.51	0.48	0.03
							(0.23)
B. Alphas on three factors plus IVOL factor							
Low	Small	–0.09%	–0.05%	–0.12%	–0.02%	–0.06%	–0.16%
		(1.04)	(–0.60)	(–1.60)	(–0.27)	(–0.86)	(–1.26)
Low	Big	0.27**	–0.11	–0.08	–0.28***	–0.25**	–0.52**
		(2.39)	(–1.23)	(–0.78)	(–2.61)	(–1.96)	(–2.51)
Medium	Small	0.27**	0.17*	0.27***	0.20**	0.10	–0.17
		(2.27)	(1.80)	(3.35)	(2.40)	(1.04)	(–1.16)
Medium	Big	0.52***	0.46***	0.15	0.03	–0.08	–0.61**
		(2.98)	(3.36)	(1.44)	(0.26)	(–0.55)	(–2.19)
High	Small	–0.30**	–0.37***	–0.26**	–0.41***	–0.46***	–0.16
		(–1.97)	(–3.12)	(–2.40)	(–3.87)	(–4.00)	(–0.79)
High	Big	–0.41	–0.13	0.35	–0.03	0.07	–0.48
		(–1.35)	(–0.50)	(1.24)	(–0.12)	(0.28)	(–1.11)
Average		0.07	–0.01	0.05	–0.09	–0.35	–0.19
							(–1.15)
Average excluding high and big		0.17	0.02	–0.01	–0.10	–0.32	–0.32**
							(–2.14)

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

positive for the low-IVOL portfolios and negative or zero for the high-IVOL portfolios, there is no difference between the returns on the high-β_{IVOL} and low-β_{IVOL} quintiles. Panel B of Table 6 reports alphas on the three factors plus the IVOL factor. Consistent with the predictions of the characteristic model (Daniel and Titman 1997, 1998), alphas tend to be significantly positive for the low-β_{IVOL} portfolios and negative for the high-β_{IVOL} portfolios. The rightmost column reports alphas on zero-investment "characteristic-balanced" portfolios that are constructed to have approximately equal IVOL and ME. In the bottom-right corner, we report alphas on the "combined portfolio," which is an equal-weighted combination of the six portfolios. If the IVOL effect is due to the IVOL characteristic, the intercepts should be significantly negative, indicating that the factor adjustment tends to overestimate the returns on high-β_{IVOL} stocks and underestimate the returns on low-β_{IVOL} stocks. That is, in fact, what the results in the rightmost column of Panel B show: All alphas are negative—two of them significantly so—and the alpha on the combined portfolio is –0.19% (t = –1.15), or –0.32% (t = –2.14) after excluding the high and big group, which has an average of only 12 stocks in each quintile.

Figure 2 plots the cumulative monthly returns on the characteristic-balanced portfolio and the factor-balanced portfolio, constructed as in Daniel and Titman (1998) by going long the factors in the loadings on the characteristic-balanced portfolio and going short the characteristic-balanced portfolio (the factor loadings on the factor-balanced portfolio are zero). Figure 2 shows the value of the portfolios over time. The characteristic-balanced (CB) portfolio, which should offer a high average return if the factor model is correct (i.e., if IVOL is associated with systematic risk), has an average return of zero in the earlier period but has negative returns in the more recent period. In contrast, the factor-balanced (FB) portfolio has a high average return over the full period. Because the CB portfolio's loadings on the factors are low (and assumed constant over the period), the variability of the FB portfolio is similar to that of a short position on the CB portfolio. Nevertheless, the cumulative return on the FB portfolio is significantly larger than the absolute loss on the CB portfolio. We conclude that the evidence supports a characteristic-based mispricing interpretation of the "IVOL" anomaly, rather than one based on systematic risk.

Figure 2. Cumulative Returns on Characteristic- and Factor-Balanced Portfolios, 1966–2011

To summarize, researchers have identified prior stock return idiosyncratic volatility as a surprisingly reliable predictor of returns beyond size and book-to-market effects. Taken together, our research findings suggest that the previously identified excess returns on low-IVOL stocks do not arise from the correlations of those stocks with some pervasive (systematic) factor. Instead, our results indicate that the abnormal returns on low-IVOL stocks most likely arise from market mispricing associated with certain characteristics of low-volatility companies; that is, investors appear to prefer high-volatility stocks to low-volatility stocks. Our empirical findings provide additional support for those who conjecture that the low-risk anomaly emanates from investor preferences having to do with behavioral considerations (Baker et al. 2011) and/or from limits to effectively arbitraging away any mispricing (Li et al. 2014). We encourage further research to disentangle the underlying sources of excess returns.

Conclusion

Contrary to fundamental expectations, researchers have found that a strategy of buying previously low-volatility stocks and selling previously high-volatility stocks has historically generated substantial abnormal returns in US and international markets. By asking whether there are pervasive systematic factors (and thus risk premiums) that are directly associated with low-volatility companies, we sought to answer a fundamental question concerning the so-called low-volatility anomaly.

Our analysis offers important insights into whether the anomalous low-risk effects are driven by systematic risks or market mispricing. The asset-pricing literature provides diagnostic methods for evaluating the source and mechanisms that drive a particular anomalous effect. We used these descriptive procedures to examine whether the return patterns of volatility characteristic–sorted portfolios are consistent with a factor model (suggesting systematic risk) or with market mispricing.

Our results indicate that market mispricing best characterizes the link between low volatility and future returns, which suggests that the high anomalous returns of low-volatility portfolios identified in the literature cannot be viewed as compensation for some hidden factor risk. Thus, investors appear to prefer high-volatility stocks to low-volatility stocks.

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Notes

1. Recent research has questioned the existence of the negative relationship between idiosyncratic volatility and subsequent returns as reported in Ang et al. (2006, 2009). For example, Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) showed that the return association is mostly due to how Ang et al. (2006) measured idiosyncratic volatility and that the approach of Ang et al. essentially captures a large return reversal effect. Fu (2009) also showed that the idiosyncratic volatility forecast from an EGARCH (exponential generalized autoregressive conditional heteroskedastic) model is significantly positively related to subsequent returns. Finally, through a variety of different measures of idiosyncratic-volatility, Bali and Cakici (2008) showed no significant relationship between idiosyncratic volatility and expected returns.
2. Cowan and Wilderman (2012) offered an intriguing risk-based explanation—namely, that low-risk stocks trade at a premium to high-risk stocks owing to the asymmetry in returns during both up markets and down markets. They suggested that low-beta stocks, versus their high-risk counterparts, provide essentially equivalent downside market protection but much less upside potential—that is, high-beta stocks provide more upside potential but suffer roughly in line with low-beta stocks in market downdrafts, and so low-risk stocks must offer an additional expected return to entice investors to participate.
3. These methods use cross-sectional tests that combine both characteristic and factor modeling. Pure factor analysis identifies time-series covariations in returns between the factors under study but does not allow inferring the source of those returns; cross-sectional analysis seeks to reveal characteristics, or attributes, that correspond to those returns.
4. We estimated market (or CAPM) betas by using the market model in which the dependent variable is company-level monthly excess stock returns and the market index is the CRSP value-weighted index over the prior 36 months (a minimum of 24 months).
5. See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
6. We obtained the Fama–French factors ($R_M - R_f$, SMB, and HML) and the risk-free rate from Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).
7. The average monthly number of stocks in each quintile portfolio as we move from the top to the bottom of Table 5 is 155, 102, 205, 51, 246, and 12.

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